Junior Solutions - TD Canada Trust High School Mathematics Competition 2012

Multiple Choice Questions.

1. B.
2. B.
3. E.
4. C.
5. C.
6. C.
7. B.
8. A.
9. A.
10. A.
11. D.
12. E.
13. A.
14. D.
15. B.
16. B.
17. D.
18. B.
19. C.
20. A.

Full Solution.

1. As $v \geq 1$ and $w \geq 1$, we see that $v + \frac{1}{w+1} > 1$, hence $0 < \frac{1}{v+\frac{1}{w+1}} < 1$. Therefore, u is the integer part of $23/7$, namely 3. Thus, $\frac{1}{v+\frac{1}{w+1}} = 2/7$, and hence $v + \frac{1}{w+1} = 7/2$. Since $w \geq 1$, $0 < \frac{1}{w+1} < 1$, and hence v is the integer part of $7/2$, namely 3. It now follows that $\frac{1}{w+1} = 1/2$, and therefore $w = 1$.
2. Since each side is nonnegative, let us square both sides. We obtain $x^2+2xy+y^2 > 1+2xy+x^2y^2$. Rearranging, this is $1 - x^2 - y^2 - x^2y^2 < 0$. That is, $(1 - x^2)(1 - y^2) < 0$. Therefore, either $1 - x^2$ is positive and $1 - y^2$ is negative, or vice versa. But x is an integer. Therefore, if $1 - x^2$ is positive, then $x = 0$. Similarly, if $1 - y^2$ is positive, then $y = 0$. Either way, $xy = 0$. To see that this is indeed a valid solution, use $x = 0$, $y = 2$.