Canada Trust

Lakehead
UNIVERSITY

Department of Mathematical Sciences

March 21, 2013   HIGH SCHOOL MATHEMATICS COMPETITION

SENIOR COMPETITION
Grades 11 and 12

Name:

E-Mail:

School & Grade:

Telephone:

Question #	*Your Answer*	For Markers Use only
1		/5
2		/5
3		/5
4		/5
5		/5
6		/5
7		/5
8		/5
9		/5
10		/5
11		/5
12		/5
13		/5
14		/5
15		/5
	Number of Unanswered Questions	x 1
		/75

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Place all answers in the multiple choice boxes on the front page of the answer booklet.

Each question is worth:

 5 marks for a correct answer
 1 mark for a blank answer
 0 marks for an incorrect answer.

(1) If $f(\sqrt{1+x}) = (1+x)^{3/2}$, then $f(2) =$

 (A) 1   (B) 4   (C) 8   (D) 12   (E) none of these

(2) How many ways can we arrange three As, four Bs and two Cs so that what we get reads the same forwards and backwards?

 (A) 0   (B) 9   (C) 10   (D) 12   (E) 24

(3) For how many values of x (in radians), with $-\pi \leq x \leq \pi$, do we have $\sin^2(x) = 0.5\sin(2x)$?

 (A) 0   (B) 2   (C) 4   (D) 5   (E) none of these

(4) The remainder when $x^2 + bx + 3$ is divided by $x - 1$ is twice the remainder when it is divided by $x + 1$. What is b?

 (A) 0   (B) 3/4   (C) 4/3   (D) 2/3   (E) 2/5

(5) For any positive integer n, write $n! = n(n-1)(n-2)\cdots(2)(1)$. What is the smallest positive integer k such that k times 11! is a perfect square?

 (A) 11   (B) 77   (C) 2310   (D) $(2310)^2$ (E) 11!

(6) Let $f(x)$ be a polynomial of degree 3 having leading coefficient 1. If $f(1) = 10$, $f(2) = 20$ and $f(3) = 30$, what is $f(0)$?

 (A) -10 (B) -6 (C) 0   (D) 1   (E) 5

(7) Given a rectangle ABCD, a point E is chosen on AD such that BC has length 5, BE has length 4 and EC has length 3. What is the perimeter of ABCD?

 (A) 12/5   (B) 20/3   (C) 12   (D) 74/5   (E) 24

(8) What is $\sin(1°) + \sin(2°) + \cdots + \sin(359°)$?

 (A) 0   (B) $\pi/2$ (C) π (D) $360°$ (E) none of these

(9) A point E lies inside the rectangle ABCD. The distance from A to E is 3, the distance from B to E is 4 and the distance from C to E is 5. What is the distance from D to E?

 (A) 4   (B) 6   (C) $\sqrt{12}$ (D) $\sqrt{15}$ (E) $\sqrt{18}$

(10) What is $\sin(15°)/\sin(75°)$?

 (A) $\pi/2$ (B) $\sqrt{3}/2$ (C) $(1-\sqrt{2})/\sqrt{3}$ (D) $(\sqrt{3}-1)/(\sqrt{3}+1)$ (E) ∞

(11) Find the coefficient of y when the following polynomial is expanded:
$$(\cdots(((y-2)^2 - 2)^2 - 2)^2 - 2 \cdots)^2$$
where there are 100 instances of "-2".

(A) -4 (B) -2^{100} (C) -2^{198} (D) -2^{199} (E) -2^{200}

(12) What is the area of the region in the xy-plane described by the inequality $|x+y| + |x-y| \leq 2$?

(A) $1/2$ (B) $1/\sqrt{2}$ (C) 1   (D) 2   (E) 4

(13) If x, y and z are positive integers with $xyz = 3125$ and $x + y + z = 151$, what is the largest of x, y and z?

(A) 25   (B) 51   (C) 125   (D) 149   (E) none of these

(14) If $\sin(x) + \cos(x) = -1/3$, what is $\sin^3(x) + \cos^3(x)$? (Here, x is measured in radians.)

(A) $-13/27$ (B) $-4/9$ (C) $-1/27$ (D) $26/27$ (E) impossible to determine

(15) How many distinct real roots does the polynomial $x^5 + 4x^4 + 7x^3 + 7x^2 + 4x + 1$ have?

(A) 0   (B) 1   (C) 2   (D) 3   (E) more than 3