Canada Trust

Lakehead
UNIVERSITY
Department of Mathematical Sciences

April 25, 2012 HIGH SCHOOL MATHEMATICS COMPETITION

JUNIOR COMPETITION
Grades 9 and 10

Name: ______________________

E-Mail: ______________________

School & Grade: ______________________

Telephone: ______________________

Question #	*Your Answer*	For Markers Use only
1		/4
2		/4
3		/4
4		/4
5		/4
6		/4
7		/4
8		/4
9		/4
10		/4
11		/4
12		/4
13		/4
14		/4
15		/4
16		/4
17		/4
18		/4
19		/4
20		/4
	Number of Unanswered Questions	x 1
		/80

Full Solution For Markers use (full solution):	
Question #	Mark
1	/10
2	/10
Full Solution Total	/20

Instructions for full solution questions:

- Place your solutions to these questions in this answer booklet.
- If you require additional space, use the back of the page but leave a note indicating this to the marker.

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Multiple Choice (80 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-20 below are worth:

4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) The value of $3^2 \cdot 9^4 \cdot 27^3$ is

(A) 3^9 (B) 3^{19} (C) 3^{24} (D) 3^{144} (E) none of these

(2) When $(1+x^2)(1+x^3)(1+x^5)(1+x^7)$ is expanded, the coefficient of x^8 is

(A) 0 (B) 1 (C) 2 (D) 24 (E) none of these

(3) Let $a = 2^{555}$, $b = 3^{444}$ and $c = 5^{222}$. Which of the following is true?

(A) $a < b < c$ (B) $c < b < a$ (C) $b < c < a$ (D) $b < a < c$ (E) $c < a < b$

(4) We have a group of 60 students, each of whom is either wearing running shoes or sandals. Given that (i) there are 12 boys wearing running shoes, (ii) there are 34 girls and (iii) there are 27 people wearing sandals, the number of girls wearing sandals is

(A) 0 (B) 12 (C) 13 (D) 14 (E) 21

(5) In how many ways can we arrange three As and eight Bs so that what we get reads the same forwards and backwards?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

(6) Five triangles have the side lengths listed below. Which one has the greatest area?

(A) 10, 24, 24 (B) 10, 24, 25 (C) 10, 24, 26 (D) 10, 24, 27 (E) 10, 24, 33

(7) What is the remainder when 2^{12} is divided by 7?

(A) 0 (B) 1 (C) 2 (D) 3 (E) none of these

(8) A triangle has integer side lengths 15, u and v with $uv = 105$. The perimeter of the triangle is

(A) 37 (B) 41 (C) 45 (D) 53 (E) 120

(9) If we multiply 201220122012 by 999999999999, how many times does the digit 4 appear in the answer?

(A) 0 (B) 1 (C) 4 (D) 6 (E) 12

(10) How many polynomials $f(x)$ are there with integer coefficients such that $f(2) = 4$ and $f(4) = 5$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

(11) Suppose we let $s_1 = 4$, $s_2 = 8$, $s_3 = 12$ and for all $n \geq 4$, let $s_n = \frac{(n+2)s_{n-3}}{n-1}$. Then s_{2012} is

(A) $\frac{2014}{2011}$ (B) 2014 (C) 4024 (D) 4028 (E) 6072

(12) Consider all of the three-digit numbers formed using only the digits in $\{1, 2, 3\}$. (Repetitions are allowed, so 313 is a valid number.) The sum of these numbers is

(A) 27 (B) 1332 (C) 1998 (D) 4995 (E) 5994

(13) For any real number x, let $\lfloor x \rfloor$ denote the largest integer that is less than or equal to x. (For instance, $\lfloor 3.8 \rfloor = 3$.) For how many positive real numbers x is $x^{\lfloor x \rfloor} = 11$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

(14) If we let $a \circ b = \frac{a-b}{b}$, then $6 \circ (3 \circ 2) =$

(A) $-1/2$ (B) 0 (C) $1/2$ (D) 11 (E) 36

(15) Let $s_1 = 5$. For each $n \geq 2$, if s_{n-1} is even, then $s_n = \frac{s_{n-1}}{2}$. If s_{n-1} is odd, then $s_n = 3s_{n-1} + 1$. Then s_{2012} is

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

(16) How many of the integers from 1 to 300 do not contain the digit 3?

(A) 240 (B) 242 (C) 243 (D) 270 (E) none of these

(17) How many pairs of positive integers (x, y) are there such that $xy - 5x - 5y = 1$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 5

(18) Violet, Wilbur, Xena, Yolanda and Zeke are considering going to a party. If Violet goes, then Wilbur will go too. Only one of Wilbur or Xena will go. Xena and Yolanda will either both go or both not go. At least one of Yolanda and Zeke will go. If Zeke goes, then both Violet and Yolanda will go. How many of these five people go to the party?

(A) 1 (B) 2 (C) 3 (D) 4 (E) this is impossible

(19) How many nonempty subsets S of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ satisfy the following property: if a is in S, then $8 - a$ is in S?

(A) 4 (B) 8 (C) 15 (D) 16 (E) 21

(20) Consider a regular 16-gon. How many different rectangles can we obtain by using 4 vertices from the 16-gon?

(A) 28 (B) 30 (C) 224 (D) 240 (E) 1820

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Full Solutions (20 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. Suppose that u, v and w are positive integers such that

$$u + \frac{1}{v + \frac{1}{w+1}} = \frac{23}{7}.$$

Find u, v and w, and explain why the values you found are the only values that work.

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2. Let x and y be integers such that $|x+y| > |1+xy|$. Find all possible values of the product xy, and explain why these are the only values possible.