Challenge Problems:

(1) Find all triples of real numbers (a, b, c) such that $a + b + c = ab + ac + bc = 3$.

(2) Consider a cube with each edge having length 1. Form a regular octahedron whose vertices are the centres of the faces of the cube. What is the volume of this octahedron?

(3) Find a nonzero polynomial with integer coefficients whose roots include $\sqrt{5} + \sqrt{2}$ and $\sqrt{5} - \sqrt[3]{2}$.

(4) Let D_1 and D_2 be unit disks in the plane. Suppose the boundary circle of D_2 passes through the centre of D_1. What is the area of the union, $D_1 \cup D_2$?

(5) Find all triples of integers (a, b, c) such that $a^2 + b^2 + c^2 = 2abc$.

(6) We have a group of 17 people. Assume that for each pair of people, they either love each other, hate each other, or don't know each other. Show that there are three people in the group who all love each other, all hate each other, or all don't know each other.