Team Members: ______________________

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1. Define a sequence $\{a_n\}$ by $a_1 = 123342$ and for each n, a_{n+1} is the sum of the cubes of the digits of a_n. What is a_{2011}?

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2. A triangle lies inside a rectangle. Prove that the perimeter of the triangle is smaller than the perimeter of the rectangle.

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3. Show that if $n > 2$, then $2^n - 1$ and $2^n + 1$ cannot both be prime.

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4. Define a sequence $\{a_n\}$ by $a_1 = 1$, $a_2 = 2$, and $a_{n+2} = a_{n+1} + 2a_n$ for all $n \geq 1$. What is $a_{1000} + a_{999}$?

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5. Solve the equation $(\sqrt{x})^{\log_{10} x} = 10$ for x.

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6. Given that x and y satisfy $x + y = 12$ and $x^3 + y^3 = 36$, what is $x^2 + y^2$?

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7. An ant starts out at $(0, 0)$ facing in some direction. Each second it walks forward one unit, and then turns clockwise by 60°. What is the perimeter of the shape the ant has traced out when it returns to its starting point?

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8. Find all solutions in positive integers x and y to the equation $2^{x+2} + 2^x = 5^{y+1} - 5^y$.

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9. Three unit circles are mutually externally tangent. A fourth circle passes through the centres of all three. What is the radius of this fourth circle?

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10. On Anthrax Island, there are 42 red chameleons, 49 blue chameleons and 59 green chameleons. Whenever two chameleons of different colours meet, they will both immediately change to the third colour. Is it possible that all of the chameleons on the island will ever be of the same colour?