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1. If $\log_5 x = 2 + \log_5 y$, what is the value of x/y ?

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2. If the graph of $y = m^x$ passes through $(2, 3)$ and $(3, k)$, what is the value of mk ?

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3. Given that a and b are positive real numbers with $a + b = 6$, what is the minimum possible value of

$$(1 + 1/a)(1 + 1/b)?$$

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4. Let ABCD be a square. Take a point E on side BC and a point F on side CD such that AEF is a triangle each side of which has length 10. Find the side length of the square.

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5. Recall that an integer $n > 1$ is said to be prime if whenever $n = ab$, for positive integers a and b , either $a = 1$ or $b = 1$. For a certain integer c , $y = cx^2 + 19x$ passes through two points with positive integer coordinates, such that the y -coordinate of each point is prime. Find c .

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6. Recall that for a positive integer n , $n! = n(n-1)(n-2)\cdots(2)(1)$. Find the largest prime factor of $(25!)^3 - (24!)^3$.

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7. Suppose that $f(1+x) = f(1-x)$ for all real numbers x . If f has exactly 8 real roots, find the sum of these roots.

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8. There is a band consisting of n pirates. They are ranked $1, 2, 3, \dots, n$, with 1 being the leader and n having the lowest rank. They must divide one million dollars worth of treasure in the following way. The leader proposes a dollar amount for each pirate (with zero for some being a possibility). All of the pirates, including the leader, vote. If 50% or more approve, then the leader's plan is adopted, and we are done. Otherwise, the leader must resign from the band forever, and everybody else moves up one rank (so that the old number 2 becomes the new leader). This is repeated until a plan is finally adopted.

You may assume that the pirates are all greedy; that is, the leader will give himself as much as he thinks he can get, and the others will happily vote against a plan if they think they can get \$1 more by doing so. However, if they think they will get the same amount by voting down a plan, then they will vote in favour of it to avoid forcing the leader to quit. Also, all of the pirates are very good at logical reasoning.

Based on these rules, how much will each pirate get if (a) $n = 2$, (b) $n = 3$, (c) $n = 4$?

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9. Bob and June together owned n hockey cards. They sold them all, receiving n dollars for each card. The buyer paid them in \$10 bills as far as possible, with the remaining amount (less than \$10) paid in loonies. They divided the bills as evenly as possible, but there were an odd number, and June received one more bill than Bob. Bob took all of the loonies. How much money does Jill owe Bob if they are to come out even? (Hint: You can't work out n exactly. But start by considering the remainder when n is divided by 10.)

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10. The small circle has radius r while the larger circle has radius $2r$. The larger circle passes through the centre of the smaller one. Denote by O one of the points where the two circles intersect, and let A be as shown (where the line passes through the centres of both circles). Draw the circle with center O and radius \overrightarrow{OA} . What is the area of the circle you have drawn?

