<u>**2006 Thunder Bay High School Mathematics Competition**</u>

<u>**Senior Team Answers**</u>

For complete solutions to any of the problems or for discussions of any of the problems, email ryanholm72@hotmail.com.

Note to markers: For the full solution questions students are expected to provide much more detail than I have here to receive full marks; this is just a reference sheet.

Note to students: The solutions provided here are only *sketch solutions*. Steps have been omitted and the solutions are meant to guide any student who has made a valiant attempt at solving the problem.

<u>Full Solution (Answers and sketch solutions):</u>

1. 6, 15. Drawing a picture of each situation, it is clear that whenever you have drawn n distinct lines, it is possible to draw another line intersecting each of these exactly once and it is not possible to get more intersections as each line can intersect each other line at most once. Hence, for four lines there are a maximum of 1 + 2 + 3 = 6 intersections, and 1 + 2 + 3 + 4 + 5 = 15 for the case of 6 distinct lines.

2. 9. There are numerous systematic ways of counting all of these.

3. No. Suppose he were to put 1 in the first bag, 2 in the second bag, 3 in the third bag, …, 10 in the tenth bag. Then he would need a total of 1 + 2 + 3 +…+ 10 = 45 hamburgers.

4. 25, 2500, 250000. The sum of the odd integers between 0 and 10^n will be the sum of all the integers between 0 and 10^n minus the sum of the even integers between 0 and 10^n. Using the standard formula $1+2+3+...m=\frac{m(m+1)}{2}$ we calculate the desired sums, since the sum of the even integers from 2 to $2n$ will be 2(1 + 2 + 3 + … + n) = $n(n+1)$ (by using the formula).

5. $\frac{N^3}{M^2}$. Since M men working M^2 hours produce M articles, one man working M^2 hours produces one article. Therefore, one man produces $\frac{1}{M^2}$ of one article in one hour, so N men working N hours a day for each of N days will produce $N\times N\times N\times\frac{1}{M^2}=\frac{N^3}{M^2}$ articles.

6. 4356. This question can be solved using the quadratic formula and then working out some messy expressions. However, for a clever solution, notice that the since $(x-a)(x-b)=x^2-(a+b)x+ab=x^2+11x+6$, we have that $a+b=-11$, $ab=6$, so that $a^2\times b^2\times(a+b)^2=6^2\times(-11)^2=4356$.

7. 3, 3, 8. Listing all the twelve decompositions of 72 into three facts and noting the sum of the factors, there is only one sum occurring twice, namely $14=2\times6\times6=3\times3\times8$ (and this is why the guest had noticed that originally the problem was indeterminate). Since the two youngest are twins, the desired decomposition is the latter.

8. 0. Whenever 2005 of the letters are in their correct envelopes, the other letter must necessarily be in its correct envelope. Hence, it is never possible to have *exactly* 2005 of the letters in their correct envelopes.

9. 3, 2, \$5.11. We must have that _679_ is divisible by 72. So it must be divisible by both 8 and 9. If it is divisible by 8, the number 79_ must be divisible by 8 (since 1000 is divisible by 8) and so 79_ must be 792. If _6792 is divisible by 9, the sum of its digits must be divisible by 9 and so the first faded digit must be 3. Finally, $\$367.92 \div 72 = \$5.11$.

10. $x = 5, y = 11$ or $x = 11, y = 5$. Let $a = x + y$ and $b = xy$. Hence, the given equations become $a + b = 71$ and $ab = 880$. Solving this system of equations gives us that $a = 16, b = 55$ or $a = 55, b = 16$. Thus, either $x + y = 16$ and $xy = 55$, or $x + y = 55$ and $xy = 16$. Only the latter system has integral solutions, namely $x = 5, y = 11$ or $x = 11, y = 5$.

11. 15, 20, 25. Let a, b, c denote the sides of the triangle with the latter being the hypotenuse. We are given

(1) $a + b + c = 60$

(2) $a^2 + b^2 = c^2$

(3) $\frac{ab}{2} = \frac{12c}{2}$,

(1) following from the perimeter, (2) following since the triangle is a right triangle, and (3) following from calculating the area of the triangle in two different ways. We have the identity $(a+b)^2 = a^2 + b^2 + 2ab$ and substituting (1), (2), (3) into this identity gives us $(60-c)^2 = c^2 + 24c$. Solving this gives $c = 25$ from which the result follows.

12. If s is a number in the sequence, then s must have the form 11 + 100m, where m is a non-negative integer. Now 11 + 100m = 4(25m + 2) + 3. In particular, s has a remainder 3 when divided by 4. But notice that any square must be of the form $4n^2$ or $4n^2 + 4n + 1$ for some integer n and hence must leave a remainder of 0 or 1 when divided by 4. Indeed, any integer is of the form $2n$ or $2n+1$ for some n (since any integer is either even or odd) and $(2n)^2 = 4n^2$, $(2n+1)^2 = 4n^2 + 4n + 1$.