TD Canada Trust

THUNDER BAY HIGH SCHOOL MATHEMATICS COMPETITION

SENIOR TEAM COMPETITION

Grades 11 and 12

Wednesday, May 10, 2006
1:00pm – 2:30pm

also sponsored by

BMO Nesbitt Burns Private Client Division · NSERC CRSNG · Thunder Bay Ventures Community Futures Development Corporation · Lakehead University Department of Mathematics · Canadian Mathematical Society

Instructions:

- You are to work with other competitors from your school in teams of at most three.
- Do not begin until you are instructed to do so.
- Fill in all required information on the front page of your team's answer booklet.
- ***Calculators are not permitted***.
- Rulers, compasses, protractors, rough paper and graph paper are permitted.
- Diagrams are not drawn to scale.
- Y*ou must place* ***all*** *of your answers in the answer booklet.*

Scoring:

- This team competition is out of 120 marks.
- There are 12 full solution questions.

Full Solution (120 marks):

- Each question is worth 10 marks.
- Sufficient *work must be shown* to receive full marks.
- Partial credit may be given to incomplete solutions if relevant work is shown.

<u>Full Solution (120 Marks)</u>

Place your solutions to these questions in the answer booklet.

Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So ***show your work***!

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1. What is the maximum number of points of intersection of 4 distinct straight lines? How about 6 six distinct straight lines?

2. Find the number of different ways 24 cents can be made using Canadian coins.

3. Mike Montanaro has 10 bags and 44 hamburgers. He wants to put his hamburgers into his bags so that each bag contains a different number of hamburgers. Can he do this?

4. Find the sum of the odd integers between
   (i) 0 and 10
   (ii) 0 and 100
   (iii) 0 and 1000

5. If M men working M hours a day for each of M days produce M articles, then how many articles (not necessarily an integer) are produced by N men working N hours a day for each of N days?

6. Let a and b be the roots of the equation $x^2 + 11x + 6 = 0$. Find the value of $a^2 \times b^2 \times (a+b)^2$.

7. "How many children have you, and how old are they?" asked the guest. "I have three boys," said Mr. Harris. "The product of their ages is 72 and the sum of their ages is the street number of my house." The guest went to look at the entrance, came back and said to Mr. Harris, "The problem is indeterminate." "Yes, you're right," replied Mr. Harris. "Did I mention the two youngest are twins?" How old are the three boys?

8. After a typist had written 2006 letters and had addressed the 2006 corresponding envelopes, a careless mailing clerk inserted the letters in the envelopes at random, one letter per envelope. What is the probability that *exactly* 2005 letters are inserted in the proper envelopes?

9. Among grandfather's papers a bill was found that read:

   72 turkeys for exactly $_67.9_

   The first and last digit of the number that represented the total price of the turkeys had faded away and are now illegible. What are the two faded digits and what was the price of one turkey assuming that each turkey is the same price?

10. Let x and y be positive integers. Find the solution to the following system of equations:

$$xy + x + y = 71 \qquad \text{and} \qquad x^2 y + xy^2 = 880.$$

11. The length of the perimeter of a right angle triangle is 60 metres and the length of the altitude perpendicular to the hypotenuse is 12 metres. What are the lengths of the sides of the triangle?

12. Prove that no number in the sequence 11, 111, 1111, 11111, … is the square of an integer.
