

Team Members: \_\_\_\_\_

School: \_\_\_\_\_

1. What is the largest number of pigeon holes that can be occupied by 100 pigeons if each hole is occupied, but no two holes contain the same number of pigeons?

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2. Evaluate  $(25 + 5\sqrt{20})^{1/3} + (25 - 5\sqrt{20})^{1/3}$ .

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3. The lengths of the sides of a triangle are consecutive integers, and the largest interior angle is twice the smallest. What is the cosine of the smallest interior angle?

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4. Prove that  $x^n + y^n = z^n$  has no solutions in positive integers with  $z < n$  and  $n > 2$ .

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5. Show that for any positive real numbers  $a$ ,  $b$ , and  $c$ , we have

$$(a+b)(a+c) \geq 2\sqrt{abc(a+b+c)}$$

Hint: First show that for any nonnegative real numbers  $x$  and  $y$ , we have

$$(x+y)/2 \geq \sqrt{xy}.$$

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6. One hundred students, no two of whom have the same height, are arranged in desks for a test. The desks are laid out in a ten by ten grid. We select two students from this group as follows. First we select the tallest student in each row in our grid, and from this group of ten we take the shortest. Call this student  $A$ . Next, with the students back in their original positions, we select the shortest student in each column, and from this group of ten, we take the tallest. Call this student  $B$ . Assume that  $A$  and  $B$  are not the same person. Which student is taller,  $A$  or  $B$ ?

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7. We have a circle of radius 1. On the circle we have six evenly spaced points. We draw inside the circle a six-pointed star with these points as vertices, as shown. What is the area of the star?

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8. A rectangular water tank is 10 metres by 10 metres on the end, and 20 metres long. Its top side is open. We tip the tank, keeping one long edge on the ground, until the upper edge is 8 metres from the ground, as shown. How much water can we put in the tank in this position without spilling?



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9. From a square, remove an isosceles right triangle from each corner as shown, in such a way as to leave a rectangle. Given that the area removed is 32, what is the length of the line segment  $a$ ?

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10. We have three round clocks sitting next to each other on a shelf as shown. Each of these clocks has no numbers and has only an hour hand, and these all point to twelve. We (quickly, so that no time passes) roll one clock over the other two, without slipping, as shown. What hour does this clock now show?