Senior Individual Solutions - TD Canada Trust High School Mathematics Competition 2010

Multiple Choice Questions.

1. A.
2. B.
3. D.
4. B.
5. D.
6. D.
7. D.
8. B.
9. E.
10. B.
11. D.
12. C.
13. A.
14. D.
15. D.
16. A.
17. B.
18. A.
19. B.
20. B.

Full Solution.

1. We have $c^2 + ac + b = 0$ and $c^2 + bc + a = 0$. Subtracting, we get $(a - b)c + (b - a) = 0$. Since $a - b \neq 0$, we divide through by $a - b$ and get $c - 1 = 0$. Thus, $c = 1$ and part (C) is false. Substituting $c = 1$ into either of our original equations, we get $1 + a + b = 0$; thus, (A) and (B) are both true. Since $c = 1$, (D) amounts to $a + 1 = -b$, which is equivalent to (B). Therefore, only (C) is false.
2. We have $f(2) = f(1+1) = f(1) + f(1) + 3(1)(1) = 2 + 2 + 3 = 7$. Similarly, $f(4) = f(2) + f(2) + 3(2)(2) = 7 + 7 + 12 = 26$. Finally, $f(8) = f(4) + f(4) + 3(4)(4) = 26 + 26 + 48 = 100$.
3. Label the corners of the rectangle ABCD beginning in the upper left, and proceeding clockwise. Let E be the intersection point on the top side of the rectangle and F the intersection point on the bottom side. By construction, AE and FC have the same length. Thus, since we have two right triangles, and of course AD and BC both have length 6, we see that DE and FB have the same length. Clearly, then, DEBF is a parallelogram. Drop a perpendicular from F to DE, and let G be the point where it meets DE. Then the length of FG is the distance we are looking for. Clearly each of the triangles ADE and BFC has area 1/5 that of the rectangle; namely 48/5. Thus, AE and CF both have length 16/5, hence EB and DF both have length 24/5. Now, angles AED and GDF are equal, hence AED and GDF are similar right triangles. Therefore, GF/FD=AD/DE. But we have already calculated FD, and AD is given. Since we have a right triangle, the length of DE is $\sqrt{6^2 + (16/5)^2} = 34/5$. Thus, FG has length $(24/5)(6)/(34/5) = 72/17$.

   Note: It is also possible to make use of the area of the parallelogram DEBF. Its area is 3/5 of the area of the rectangle; that is, 144/5. But it is also equal to the length of DE times the length of FG. But we calculated the length of DE above, and can therefore obtain FG.