Name: ____________________

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Multiple Choice (50 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) Lisa was asked to add 12 to a certain number, and then divide the result by 4. Instead, she first added 4 and then divided by 12. She ended up with 5 as an answer. If she followed instructions correctly, what would her result have been?

(A) 5 (B) 17 (C) 20 (D) 56 (E) 60

(2) What is $(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{63} 64)$?

(A) 1 (B) e (C) 6 (D) 32 (E) 64

(3) On a sheet of paper are listed the following four statements (and nothing else).
This page contains exactly one false statement.
This page contains exactly two false statements.
This page contains exactly three false statements.
This page contains exactly four false statements.
How many of the statements are actually false?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(4) How many pairs of positive integers (a, b) are there such that $a(a+1) = 2^b$?

(A) 0 (B) 1 (C) 3 (D) 37 (E) infinitely many

(5) For any positive integer n, let $n! = n(n-1)(n-2)\cdots(2)(1)$. The last digit of $1!+2!+3!+\cdots+50!$ is

(A) 0 (B) 3 (C) 4 (D) 5 (E) 9

(6) A boat is being rolled along flat ground on perfectly round logs. If there is no slipping, and the centre of one of the logs moves three metres, then how many metres does the front of the boat move?

(A) 3 (B) 2π (C) 6 (D) 6π (E) 12

(7) We have two students, Alice and Bob. One always lies on Tuesday, Wednesday, and Thursday, and tells the truth on the other days. The other lies on Friday, Saturday, and Sunday, and tells the truth on other days. One afternoon, they make the following statements. Alice says, "I always lie on Sundays." Bob says, "I will lie tomorrow," to which Alice replies, "I will always lie on Mondays." What day is it?

(A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

(8) 10 Let $\{a_n\}$ be the sequence of integers defined by $a_1 = 2$, $a_2 = 7$, $a_3 = 4$, $a_4 = 8$, and in general, a_{n+2} equals the unit digit of $a_{n+1}a_n$. Then a_{2011} is

(A) 2 (B) 4 (C) 6 (D) 8 (E) 0

(9) How many triples (a, b, c) of positive integers are there with $a > b > c > 0$ and $1/a + 1/b + 1/c$ an integer?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

(10) What is $\sqrt{4\sqrt{7}+11} - \sqrt{7}$?

(A) $4\sqrt{7}$ (B) -1 (C) $\sqrt{7}$ (D) 2 (E) 11

Questions 11-15 below are worth:
4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(11) How many positive integers are there between 1 and 10^{2011} that have the sum of their digits equal to 2?

(A) 2011 (B) 2021055 (C) 2023066 (D) 4042110 (E) 4044121

(12) What is $\log_{10}(\tan(30°)) + \log_{10}(\tan(60°))$?

(A) 1 (B) 0 (C) e (D) π (E) $\sqrt{3}/2$

(13) What is the coefficient of n^{36} when $(n^5 + n + 1)^9$ is multiplied out?

(A) 1 (B) 9 (C) 18 (D) 36 (E) 72

(14) A lottery company issues tickets on which two distinct numbers from $\{1, 2, 3, 4, 5\}$ are chosen by the customer. The lottery company draws two distinct numbers from this list. A ticket *loses* if one or more of its numbers matches one of the numbers the lottery company chose. How many tickets need you buy in this lottery to be assured of having at least one winning ticket?

(A) 2 (B) 4 (C) 8 (D) 16 (E) no finite number will suffice

(15) The number of positive integers n for which $n^4 + n^2 + 1$ is prime is

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

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Full Solutions (30 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. Find all positive integers n such that $2n + 1$ divides $n^3 - n + 3$.

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2. Suppose that x is positive and $x^2 + 1/x^2 = 4$. Find (i) $x + 1/x$, (ii) $x^3 + 1/x^3$ and (iii) $x^5 + 1/x^5$.

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3. Let n be a positive integer. Show that if $2^n - 1$ is a prime number, then n is a prime number.