TD Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

April 30, 2008.

SENIOR INDIVIDUAL COMPETITION

Grades 11 and 12

Name: _____

School & Grade: _____

Telephone: _____

E-Mail: _____

Will you be applying to Lakehead University for September 2008? _____

Question #	*Your Answer*	For Markers Use only
1		/3
2		/3
3		/3
4		/3
5		/3
6		/3
7		/3
8		/3
9		/3
10		/3
11		/4
12		/4
13		/4
14		/4
15		/4
	Number of Unanswered Questions	x 1
		/50

For Markers use (full solution):

Question #	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Full Solution Total	/50

Name: _____

School: _____

Multiple Choice (50 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

---

(1) $\frac{1}{x}+\frac{1}{y}+\frac{1}{xy}=\frac{?}{xy}$

(A) $x+y+1$ (B) xy (C) 2 (D) 3 (E) $2x+3y$

(2) For how many positive integers n does $\sqrt{n}$ differ from $\sqrt{100}$ by less than 1.

(A) 10 (B) 38 (C) 39 (D) 48 (E) 100

(3) Define an operation $*$ on the positive integers using the rule $a*b=a^{b-1}$. What is $4*(2*3)$?

(A) 1 (B) 4 (C) 27 (D) 64 (E) 132701

(4) How many numbers from 1 to 1000 have no digits that are prime?

(A) 125 (B) 216 (C) 334 (D) 500 (E) 875

(5) Suppose a is an integer and that $\log_{13}(\log_{17}(\log_{19}(a)))=58$. How many different prime numbers are divisors of a?

(A) 0 (B) 1 (C) 3 (D) 4 (E) 6

(6) Five consecutive odd integers are added together. If the second odd number is $2m-1$, the sum of the five numbers is

(A) $10m-5$ (B) $10m-1$ (C) $10m+3$ (D) $10m+5$ (E) $10m+15$

(7) The largest value of $2-3\cos\theta$ would be:

(A) -1 (B) 0 (C) 2 (D) π (E) 5

(8) Suppose that $x^2+bx+4=0$ has exactly one solution. What is the value of b?

(A) 2 (B) -2 (C) 4 (D) -4 (E) 4 or -4

(9) A used car dealer sells two cars for $750. On the first car, he makes a profit of 50% (compared to the purchase price). For the second car, he has a loss of 40%. What is the dealer's net profit (or loss) on the transactions?

(A) -$250.00 (B) $0.00 (C) $250.00 (D) $500.00 (E) $750.00

(10) Let $f(x)=10^x$ and $g(x)=f(x+1)-f(x)$. Then $g(x)=?$

(A) 10 (B) $f(x)$ (C) $9f(x)$ (D) $10f(x)$ (E) $11f(x)$

Name: _____

School: _____

Questions 11-15 below are worth:

4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

(11) What is the largest value of $|x^2 - 9|$ for all x satisfying $|x + 3| \leq 0.1$?

(A) 0.1 (B) 0.59 (C) 0.61 (D) 3 (E) 8.99

(12) If k is a rational number such that $\sqrt[3]{9\sqrt{3} - 11\sqrt{2}} = \sqrt{3} + k\sqrt{2}$, then k equals

(A) 3 (B) -1 (C) 0 (D) 1 (E) none of the above

(13) What is the difference between $\sqrt{5 + 2\sqrt{6}}$ and $\sqrt{5 - 2\sqrt{6}}$?

(A) $2\sqrt{2}$ (B) $4\sqrt{6}$ (C) $2(6^{1/4})$ (D) $\sqrt{3}$ (E) 1

(14) Let a, b and c be numbers greater than 1. Under what circumstances is $4(\log_a(c))^2 + 3(\log_b(c))^2$ equal to $8(\log_a(c))(\log_b(c))$?

(A) always (B) if and only if $a = b$ (C) if and only if $a = b^2$
(D) if and only if $c = ab$ (E) some other conditions

(15) If $a = \frac{1}{2}$ and $(a + 1)(b + 1) = 2$, then the radian measure of $\arctan a + \arctan b$ equals

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{6}$

Name: _____

School: _____

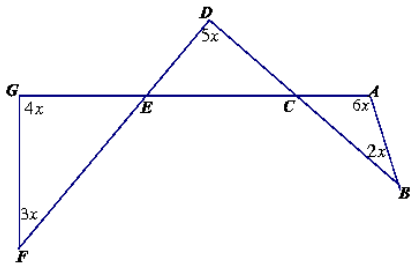
Full Solutions (50 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

---

1. In the figure shown below, what this $\angle CAB$? (Here, x is measured in degrees.)



Name: _____

School: _____

2. Let

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{2008^2}\right)=\frac{x}{2\cdot 2008}$$

What is x?

Name: _____

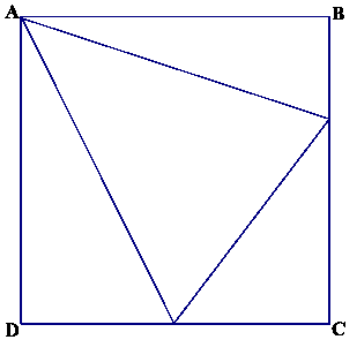
School: _____

3. Let $p(x) = 1 + a_1x + a_2x^2 + \cdots + a_nx^n$ be a polynomial where $a_1, \ldots, a_n$ are integers, and $a_1 + \cdots + a_n$ is even. Prove that there is no integer x such that $p(x) = 0$.

Name: _____

School: _____

4. Given a square ABCD with area 1, and an equilateral triangle with one vertex at A and the other two on sides BC and CD, what is the area of the triangle? (Note: diagram not to scale)



Name: _____

School: _____

5. Circles with centers A, B, and C each have radius r, where $1 < r < 2$. The distance between each pair of centers is 2. If B' is the point of intersection of circle A and circle C which is outside circle B, and if C' is the point of intersection of circle A and circle B which is outside circle C, then find the length of $B'C'$. (Note: diagram not to scale.)