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Multiple Choice (70 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) Michael had an average score of 45 on his first eight economics courses, and an average score of 41 on his first nine economics courses. What score did he receive on his ninth course?

(A) 41 (B) 9 (C) 37 (D) 33 (E) 45

(2) Determine the number of different pairs (x, y) that satisfy the system

$$\begin{cases} x + 2y &= 4 \\ x + y^2 &= 3 \end{cases}$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(3) If $f(x) = 2^x$, then 16^8 is

(A) $f(f(3))$ (B) $f(12)$ (C) $f(4^8)$ (D) $f(f(5))$ (E) $f(f(f(3)))$

(4) Suppose that $f(x) = ax + b$ where a and b are real numbers. If we have $f(f(f(x))) = 8x + 35$, what is b?

(A) 2 (B) 5 (C) 8 (D) 35 (E) 7

(5) Recall that for positive integers n, $n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$. The greatest n for which 12^n evenly divides 20! is

(A) 4 (B) 6 (C) 7 (D) 8 (E) 12

(6) If we let $a * b = \frac{a+b}{ab}$, then $4 * (3 * 3) =$?

(A) 1/12 (B) 7/8 (C) 4/7 (D) 7/4 (E) 9/4

(7) What is the last digit of $1! + 2! + 3! + \cdots + 99! + 100!$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(8) What is the value of

$$\log_2((\log_{16} 2)^{\log_2 16})?$$

(A) -4 (B) -8 (C) 0 (D) $-4\log_2 3$ (E) $-4\log_3 2$

(9) A line with slope 2 intersects a line with slope 6 at the point $(50, 30)$. What is the distance between the x-intercepts of these lines.

(A) 4 (B) 6 (C) 8 (D) 12 (E) 10

(10) Four points are located on a line. When the six distances between pairs of points are written in increasing order, they are 2,3,5,n,8,11. What is n?

(A) 5.5 (B) 6 (C) 6.5 (D) 7 (E) 7.5

Questions 11-20 below are worth:
4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

(11) Given that a, b, and c are three real numbers with $0 < a < b$ and $c \neq 0$, which of the following statements is not always true?

(A) $\frac{a}{c^2} < \frac{b}{c^2}$
(B) $ac^2 < bc^2$
(C) $a - c < b - c$
(D) $ac < bc$
(E) $a + c < b + c$

(12) What is the value of the product $\sin\frac{\pi}{16}\cos\frac{\pi}{16}\cos\frac{\pi}{8}\cos\frac{\pi}{4}$?

(A) 1/4 (B) 1/2 (C) 1/8 (D) 4 (E) 1/16

(13) What is the sum of all the different real solutions of the following equation?

$$\frac{(x^4+1)(x^6+1)}{x+1} + x - 1 = 0$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(14) Suppose, for some real numbers c and d, we have $f(x) = cx^4 + dx^2 + x + 7$. If $f(-3) = 4$, then $f(3)$ is

(A) -2 (B) 2 (C) 5 (D) 10 (E) impossible to determine

(15) What is the largest possible value of $\sin x - \cos x$?

(A) 1 (B) $1/\sqrt{2}$ (C) $\sqrt{3}/2$ (D) $\sqrt{2}$ (E) 2

(16) A square is said to be inscribed in a circle if its four corners lie on the circle. A square is said to circumscribe a given circle if its sides are each tangent to the circle (the circle fits tightly inside the square, touching all sides). Given a circle, what is the ratio of the side length of an inscribed square to that of a circumscribing one?

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{3}$ (C) $\sqrt{6}$ (D) $\frac{1}{\sqrt{3}}$ (E) $2\sqrt{2}$

(17) The value of $100^2 - 99^2 + 98^2 - 97^2 + \cdots + 2^2 - 1^2$ is

(A) 9999 (B) 5050 (C) 2525 (D) 20020 (E) 10100

(18) How many 6-digit numbers obtained by rearranging the digits in 123456 are perfect squares?

(A) 0 (B) 1 (C) 6 (D) 21 (E) 720

(19) The number of solutions (x, y, z) to the equation $x + 3y + 5z = 19$, with x, y and z positive integers, is

(A) 19 (B) 7 (C) 5 (D) 15 (E) 10

(20) Four points are located on a line. The distances between pairs of points are all distinct positive integers. From smallest to largest, these distances are $a, b, c, d, e, 10$. What is the largest possible value of $a + b + c + d + e$?

(A) 20 (B) 27 (C) 28 (D) 30 (E) 31

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Full Solutions (30 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. Let a and b be distinct integers. If the equations $x^2+ax+b=0$ and $x^2+bx+a=0$ have a common real root c, then determine which of the following statements are true. (There may be more than one; you must explain why each statement is true or false, as the case may be.)

(A) $a+b+c=0$

(B) $a+b=-1$

(C) $c=0$

(D) $c(a+c)=-b$

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2. A function f has the property that $f(x+y) = f(x) + f(y) + 3xy$ for all positive integers x, y. If $f(1) = 2$, what is $f(8)$?

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3. A rectangle with dimensions 8 cm and 6 cm is divided in three regions as in the picture. The two triangular regions have the same area while the area of the region in the middle is three times the area of each triangular region. Find the distance d between the two lines.