Senior Individual Solutions - NWO High School Mathematics Competition 2009

Multiple Choice Questions.

1. D.
2. D.
3. A.
4. E.
5. C.
6. C.
7. B.
8. E.
9. D.
10. C.
11. C.
12. E.
13. B.
14. B.
15. A.

Full Solution.

1. If Yolanda is lying, then so are Violet and Wilbur, but we can't have three liars. Thus, Yolanda is telling the truth. This means that Zora is lying, and as Xavier directly contradicts Yolanda, Xavier is lying as well. As we can only have two liars, Violet and Wilbur are telling the truth. But Violet says that either Wilbur or Xavier did it, and Wilbur says he didn't do it, so Xavier must be the criminal.
2. Let r be the original radius. Then the original circumference is $2\pi r$, so the enlarged circumference is $2\pi r + 1$. The enlarged radius is, therefore, $(2\pi r + 1)/(2\pi) = r + (1/(2\pi))$, so the radius has increased by $1/(2\pi)$ cm.
3. After each stage, the number of red balls stays odd, because we either leave the total the same, or decrease it by 2. Thus, there can never be zero red balls. The last ball must be red.
4. If the centres of the circles lie on opposite sides of the line, then clearly $D + d$ is the distance between the centres of the circles, so the smallest possible value occurs when the centre of the smaller circle lies precisely on the line. Moving the centre of the smaller circle closer to the centre of the larger circle then causes the cord to shrink (and therefore move farther away from the centre of each of the circles). Thus, $D + d$ gets larger as we move the circles closer together than the point described above. Hence, the smallest value for $D + d$ does indeed occur when the centre of the smaller circle lies on the line, and $D + d = D$. Letting C be the centre of the big circle, C' the centre of the small circle, and A a point of intersection, we see that $CC'A$ is a right triangle with legs D and r, and hypotenuse R. Thus, $D = \sqrt{R^2 - r^2}$.
5. Notice that if $ab = 100$, then $\log a + \log b = \log(ab) = 2$. Thus, $(\log 1 + \log 100) + (\log 2 + \log 50) + (\log 4 + \log +25) + (\log 5 + \log 20) + \log 10 = 2 + 2 + 2 + 2 + 1 = 9$.