2005 Thunder Bay High School Mathematics Competition

Senior Individual Answers

For complete solutions to any of the problems or for discussions of any of the problems, email ryanholm72@hotmail.com.

Note to markers: For the full solution questions students are expected to provide much more detail than I have here to receive full marks; this is just a reference sheet.

Note to students: The solutions provided here are only *sketch solutions*. Steps have been omitted and the solutions are meant to guide any student who has made a valiant attempt at solving the problem.

Multiple Choice:

1. C	9. A
2. B	10. A
3. D	11. B
4. B	12. D
5. D	13. C
6. E	14. C
7. B	15. B
8. B	

Full Solution (Answers and sketch solutions):

1. (a) 5. Two distinct lines intersect in exactly one point. A line and a circle can intersect in at most 2 two points. So there are at most 5 points of intersection and a simple picture shows that such a 5-point intersection can be achieved.
   (b) 0. Two distinct parallel lines intersect in 0 points. A line and a circle can intersect in 0 places. So there are at least 0 points of intersection and a simple picture shows that such a 0 point intersection can be achieved.

2. 10. It is easy to systematically list the 10 possible solutions. However, a more clever solution is to consider 6 letter R's written in a line. If we choose any two of the five spaces between the R's and draw a line in those two spaces, we will partition the R's into 3 groups, each group having at least 1 R. Hence, there are "5 choose 2" = $\frac{5!}{3!2!}$ = 10 ways of doing this.

3. –2. $x^3 + y^3 = (x+y)(x^2 - xy + y^2) = (x+y)[(x+y)^2 - 3xy]$. Substituting $x + y = 1$ and $xy = 1$ into this expression yields $x^3 + y^3 = -2$. Note that this can also be done by solving $x + y = 1$ and $xy = 1$ for x and y. Also note that x and y are complex numbers.

4. A > D > B > C, where A, B, C, D are the scores of Ann, Bill, Carol, and Dick, respectively. We have that:
   (I) A + C = B + D
   (II) A + B > C + D
   (III) D > B + C

Adding (I) and (II) gives that A > D. (III) shows that D > B since the scores are non-negative. Subtracting (I) from (II) gives that B > C. So combining these three inequalities gives A > D > B > C.

5. 02528. By the binomial theorem,

$$\begin{aligned}(1.0025)^{10} = (1+0.0025)^{10} &= 1+10(0.0025)+\frac{(10)(9)}{2}(0.0025)^2+\frac{(10)(9)(8)}{(2)(3)}(0.0025)^3+R \\ &= 1+0.025+0.0028125+0.000001875+R \\ &= 1.025283125+R,\end{aligned}$$

where R = $\sum_{k=4}^{10}\frac{10!}{k!(10-k)!}(0.0025)^k < (0.0025)^4\sum_{k=4}^{10}\frac{10!}{k!(10-k)!} < (0.0025)^4\sum_{k=0}^{10}\frac{10!}{k!(10-k)!}$

$$= (0.0025)^4 2^{10}$$

$$= 0.00000004\,.$$