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Multiple Choice (50 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) For what value of a does the system of equations $3x + 7y = 5$ and $x + ay = 2$ have no solutions in real numbers x and y?

(A) 2/7 (B) 5/2 (C) 1/2 (D) 7/3 (E) 3/7

(2) Given that

$$\log(x^2) - \log 7 = \log x - \log 8,$$

find x.

(A) $10^{49/64}$ (B) 49/64 (C) 64/49 (D) 7/8 (E) 8/7

(3) The real numbers x and y satisfy $2^x = 10$ and $10^y = 64$. What is the value of xy?

(A) 6 (B) 12 (C) $\sqrt{10}$ (D) $\log 2$ (E) There is more than one answer.

(4) Which of the numbers in the list: 5, 1, 1/10, 1/20, 1/40 is the largest member of the list that is smaller than $\sqrt{10020} - \sqrt{10010}$?

(A) 5 (B) 1 (C) 1/10 (D) 1/20 (E) 1/40

(5) The positive numbers a, b, c, x, y, z are such that $x/a = y/b = z/c$. The value of

$$\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$$

is

(A) 2 (B) 1/3 (C) 1 (D) 3 (E) dependent on the value of x/a

(6) If $2^{2^x} + 4^{2^x} = 42$, then what is $\sqrt{2^{2^{2^x}}}$?

(A) 2 (B) 4 (C) 8 (D) 16 (E) 65535

(7) Suppose $\sin\theta + \cos\theta = 1/3$. What is the value of $\sin(2\theta)$?

(A) 1 (B) -8/9 (C) 8/9 (D) 9/8 (E) 0

(8) Suppose a real valued function f satisfies $f(x + f(x)) = 4f(x)$ and $f(1) = 6$. What is $f(31)$?

(A) 68 (B) 72 (C) 24 (D) 6 (E) 96

(9) In my small village there is an accurate village clock which chimes once on each half hour, and n times on the n-th hour (24 times at midnight). If I can hear but not see the clock, what is the longest time I may have to wait before I know the exact time?

(A) 1 hour (B) 1/2 hour (C) 15 minutes (D) 1.5 hours (E) 45 minutes

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(10) A standard deck of cards contains fifty-two cards, four of each rank. How many possible five card hands are there that contain four cards of the same rank? (Note: order does not matter.)

(A) 526 (B) 676 (C) 624 (D) 48 (E) 13

Questions 11-15 below are worth:

4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(11) What is the area of the region of the plane determined by the inequality $2 \leq |x| + |y| \leq 3$?

(A) $\sqrt{2}$ (B) 8 (C) 10 (D) 18 (E) 5/2

(12) The sides of a triangle have lengths a, b, and c. If we have $\sqrt{a} + \sqrt{b} = \sqrt{c}$, then our triangle is:

(A) a right triangle (B) isosceles (C) acute (D) equilateral (E) nonexistent

(13) How many sets are there of two or more consecutive positive integers that sum to 100?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

(14) Sales tax is calculated at 13% and rounded to the nearest penny. How many impossible totals are there between 1 cent and $10 inclusive? (For example, 56 cents is an impossible total; indeed, a purchase of 49 cents comes to 55 cents with tax, and a purchase of 50 cents comes to 57 cents with tax.)

(A) 114 (B) 115 (C) 116 (D) 117 (E) 118

(15) Consider the right triangle drawn below. Find a.

(A) 24/5 (B) 5 (C) 7 (D) $\sqrt{10}$ (E) impossible to determine

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Full Solutions (50 Marks)
Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. Five suspects, exactly one of whom committed a crime, make the following statements.
Violet said, "Either Wilbur or Xavier did it."
Wilbur said, "I didn't do it, and neither did Zora."
Xavier said, "Violet and Wilbur are both lying."
Yolanda said, "Either Violet or Wilbur is telling the truth."
Zora said, "Yolanda is lying."
Assuming that exactly three of the suspects are telling the truth, who committed the crime?

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2. We are inflating a balloon which remains a sphere however large we inflate it. At one point, we measure the circumference of the sphere. Then we inflate it a bit more, and measure again. When we do this, we find that the circumference has increased by 1 cm. How much has the the radius of the balloon increased?

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3. A trash can contains 357 red balls and 284 green balls. Next to the trash can we have a large pile of green balls. We draw two balls at random from the trash can. If both of the balls are green, we put one back in and throw the other one away. If both are red, then we throw both of them away, and add a green ball from the pile. If one is red and the other is green, we put the red one back into the trash can and throw the green one away. We repeat this procedure until only one ball is left in the trash can. What colour is this last ball?

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4. We have two circles in the plane with radii R and r, with $R > r$. These two circles meet in two points. The line through the two points where the circles meet is at distance D from the centre of the circle of radius R, and at distance d from the centre of the other circle. What is the smallest possible value for $D + d$ (in terms of R and r)?

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5. What is the sum of the base 10 logarithms of the factors of 100 (including 1 and 100 among the factors)?