THUNDER BAY HIGH SCHOOL
MATHEMATICS COMPETITION

SENIOR INDIVIDUAL COMPETITION

Grades 11 and 12

Wednesday, May 10, 2006
9:45am – 11:00am

also sponsored by

Instructions:

- Do not begin until you are instructed to do so.
- Fill in all required information on the front page of your answer booklet.
- ***Calculators are not permitted***.
- Rulers, compasses, protractors, rough paper and graph paper are permitted.
- Diagrams are not drawn to scale.
- Y*ou must place **all** of your answers in the answer booklet.*

Scoring:

- This individual competition is out of 100 marks.
- There are 15 multiple-choice questions and 5 full solution questions.

Multiple Choice (50 marks):

- Each incorrect answer is worth 0 marks.
- Each unanswered multiple-choice question is worth 1 mark.
- Multiple-choice questions #1-10 are worth 3 marks each and #11-15 are worth 4 marks each.

Full Solution (50 marks):

- Full solution questions are each worth 10 marks.
- Sufficient *work must be shown* to receive full marks for a full solution question.
- Partial credit may be given to incomplete solutions if relevant work is shown.

Multiple Choice (50 Marks)

Place all answers in the multiple-choice boxes on the front page of the answer booklet.

Questions #1-10 below are worth: 3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer

1. The points A(1,2), B(2,3), C(-2,1), D(-1, 3), and E(2,-1) are marked on the xy-plane. The line segment that is horizontal is

   (A) AB (B) BD (C) CD (D) AE (E) CE

2. For integers *a*, *b*, and *c*, define $\|a,b,c\|$ to mean $a^b - b^c + c^a$. Then $\|1,-1,2\|$ equals

   (A) -4 (B) -2 (C) 0 (D) 2 (E) 4

3. What is the reciprocal of 2 + 1/4?

   (A) 1/2 + 4 (B) –9/4 (C) 4/9 (D) 9/4 (E) 1/2 + 2

4. If Tim runs faster than Jill, Jill runs slower than Ed, Jorge runs faster than Tim, Ed runs slower than Tim, and Pedro runs faster than Jorge, then Ed runs faster than

   (A) Tim (B) Jorge (C) Jill (D) Pedro (E) no one

5. A circle of radius 7 cm is totally contained within a square of side length 21 cm. What is the area of the square *not* covered by the circle?

   (A) 441 - π (B) 49(9 - π) (C) 49 - π (D) 441 - 3π (E) 7(3 - π)

6. The consecutive angles of a trapezoid form an arithmetic sequence. If the smallest angle is 75 degrees, then the largest angle, in degrees, is

   (A) 95 (B) 100 (C) 105 (D) 110 (E) 115

7. In the sequence

   …, a, b, c, d, 0, 1, 1, 2, 3, 5, 8, …

   each term is the sum of the two terms to its left. What is the value of a?

   (A) –3 (B) –1 (C) 0 (D) 1 (E) 3

8. Let the set consisting of the squares of the positive integers be called Ω. That is, $\Omega = \{1,4,9,16,...\}$. If a certain operation on one or more members of a set always yields a member of the set, we say that the set is closed under that operation. Then Ω is closed under

(A) addition (B) multiplication (C) division (D) subtraction (E) none of these

9. Let S be the statement

"If the sum of the digits of the whole number n is divisible by 6, then n is divisible by 6."

A value of n which shows S to be false is

(A) 30 (B) 33 (C) 40 (D) 42 (E) none of these

10. In the xy-plane, the segment with endpoints (-5,0) and (25,0) is the diameter of a circle. If the point (a, 15) is on the circle, then a is

(A) 10 (B) 12.5 (C) 15 (D) 17.5 (E) 20

Questions #11-15 below are worth: 4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer

11. Let H = { n | n is a positive integral divisor of 20}.
Let I = { m | m is an integer and the absolute value of m is less than 3}.
Which of the following sets has exactly four elements?

(A) H (B) I (C) H $\cup$ I (D) H $\cap$ I (E) H\I

12. How many positive integers exist that are less than 1000 and none of their digits is a prime number?

(A) 180 (B) 215 (C) 300 (D) 333 (E) 500

13. How many digits has the least positive multiple of 45 which contains only the digits 0 or 1?

(A) 10 (B) 9 (C) 8 (D) 7 (E) 6

14. A circle C_1 of radius r_1 and another circle C_2 of radius r_2 intersect each other in exactly one point P. Neither of the circles lies in the interior of the other. A line L is drawn that is tangent to both of these circles and L does not pass through P. If L intersects C_1 at the point P_1 and C_2 at the point P_2, the distance between P_1 and P_2 is

(A) $\sqrt{2r_1r_2}$ (B) $(\sqrt{3r_1r_2})\sqrt{2}$ (C) $2\sqrt{r_1r_2}$ (D) $r_1 + r_2$ (E) $r_1^2 + r_2^2$

15. The graph of a function y = f(x) is described as follows:
   (i) Draw a straight line from (0,0) to (2,2)
   (ii) Draw a straight line from (2,2) to (4,0)
   (iii) Draw a straight line from (4,0) to (6,2) and extend this line indefinitely.

   Which of the following equations represents this function for non-negative values of x?

   (A) $y = |x+2| + 2$ (B) $y = |x-2| + 2$ (C) $y = ||x-2| + 2|$ (D) $y = ||x+2| + 2|$ (E) $y = ||x-2| - 2|$

<u>Full Solution (50 Marks)</u>

Place your solutions to these questions in the answer booklet.

Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So ***show your work***!

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1. Being a nice brother and son, Andrew first gives half of his paycheque to his brother Ryan and then a fourth of what remains to his parents. If Andrew gave away a total of $200 to his brother and parents, how much was Andrew's paycheque?

2. A number is called a *palindrome* if it is the same regardless of whether it is read from left to right or right to left. For example, 111, 212, 14541, and 35553 are *palindromes*. Find the number of positive integers between 100 and 300 that are *palindromes*.

3. To number the pages of his functional analysis notes Ryan needs to use 1890 digits. How many pages of notes does Ryan have?

4. In the acute triangle ABC, CD is the altitude to AB, and AE is the altitude to BC. If AB = 5, CD = 3 and AE = 4, determine the length of DB.

5. If the *whatsis* is *so* when the *whosis* is *is* and the *so* and *so* is $is \times so$, what is $whosis \times whatsis$ when the *whosis* is *so*, the *so* and *so* is $so \times so$, and the *is* is 2 (*whatsis*, *whosis*, *is* and *so* are variables that are positive integers)?
