**SENIOR INDIVIDUAL COMPETITION - SOLUTIONS**
Grades 11 and 12

1. A standard calculation gives (e) $y = 3x - 1$.

2. As $n!$ is a multiple of 10 for all $n > 5$, we need only calculate $1! + 2! + 3! + 4! = 33$, so the last digit is (b) 3.

3. If she started with $x$, then $(x+4)/12 = 5$, so $x = 56$. Thus, $(x+ 12)/4$ gives us (b) 17.

4. Completing the square, we want (d) 25.

5. We have $3000/(2(10)+a) = 100$, so $a = 10$, and therefore $3000/(2(20)+ 10) = 60$, so the answer is (b) 60 000.

6. This is P(10,3) = 720, so (d).

7. As the function is zero when $x = 0$, we can rule out the first three. As it is zero when $x = 4$ as well, we can rule out the fourth one, so the answer must be (e) $||x - 2|- 2|$.

8. The statements all contradict each other, so at most one of them can be true. If all of them were false, there would be four false statements, so the last statement would in fact be true, which is impossible. Therefore the answer must be (d) 3.

9. We must have $y - x = (3x + y) - y$, so $y = 4x$. But also, $y - x = (x + 2y + 2) - (3x + y)$, so $x = 2$, hence $y = 8$, and the answer is (e) 6.

10. Since $\sin(x - y) = 0$, we must have $x - y = 0$, $\pi$ or $-\pi$. If $x - y = \pm\pi$ then we have either $(0,\pi)$ or $(\pi, 0)$, both of which work, so there are two solutions. If $x = y$, then as $\sin(2x) = 0$, we must have $x = 0$, $\pi /2$ or $\pi$. All of these solutions, $(0,0)$, $(\pi/2,\pi/2)$, and $(\pi,\pi)$ work, so there are (d) 5 solutions in total.

11. Notice that $(x + 1/x)^2 = x^2 + 1/x^2 + 2 = 6$, so since $x + 1/x$ must be positive, it is (d) $\sqrt{6}$ .

12. We want $2^{(1+3+5+...+(2n+1))/7} > 1000$. Notice that $2^{10} = 1024$, so we will look for a number close to 70 in the exponent. Now, when $n = 7$, we get $2^{64/7}$ which is just over $2^9 = 512$ and therefore less than 1000, but when $n = 8$, we get $2^{81/7} > 2^{10}$, so the answer must be (c) 8.

13. This is $(x^2 - 1)^3 = 0$. The solutions are $\pm1$, so the answer is (c) 2.

14. We have $|x^2 - 9| = |x - 3||x + 3| < (.1)(6.1) = .61$, so the answer is (e) .61.

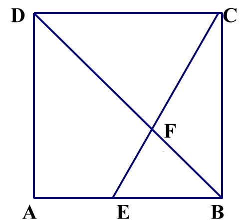
15. We have $3*3 = (3+3)/(3(3)) = 2/3$, so $6*(2/3) = 6+(2/3))/(6(2/3)) = 5/3$, and the answer is (c).

KEY:

| | |
|---|---|
| 1. | e |
| 2. | b |
| 3. | b |
| 4. | d |
| 5. | b |
| 6. | d |
| 7. | e |
| 8. | d |
| 9. | e |
| 10. | d |
| 11. | d |
| 12. | c |
| 13. | c |
| 14. | e |
| 15. | c |

Senior Proof Solutions:

1. Placing A at the origin, DB has equation $y = 1 - x$, and EC has equation $y = 2x - 1$. The point of intersection is (2/3,1/3). So the triangle has base 1/2 and height 1/3, so its area is 1/12.




2. This is $\frac{\ln 3}{\ln 2}\frac{\ln 4}{\ln 3}\cdots\frac{\ln 64}{\ln 63} = \frac{\ln 64}{\ln\ 2} = \log_2 64 = 6.$

3. We have $y = \frac{\sqrt{3} - 2 - 2\left(\sqrt{2} - 1\right)}{\left(\sqrt{3} - 1\right)^2 - 2\left(\sqrt{2} - 1\right)^2} = \frac{\sqrt{3} - 2\sqrt{2} + 1}{-2\sqrt{3} + 4\sqrt{2} - 2} = -0.5$ Thus, $[y] = [-0.5] = -1$.

4. If $n = 1$, then $2n - 1 = 1$, which is not prime, so we will assume that $n > 1$, and therefore, that $n$ is composite. Let $n = ab$, with $a > 1$ and $b > 1$. Then $2^{ab} - 1 = (2^a - 1)(1 + 2^a + 2^{2a} + \ldots + 2^{a(b-l)})$, and therefore we have a composite number, as each of the factors is larger than 1. This is a contradiction.

5. If $\cos(x) = 0$, then we would have $\sin(x) = 0$ as well, which is impossible, so let us divide by $\cos^2(x)$. Then $\tan^2(x) - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\tan(x) + 1 = 0$. Thus, $\tan(x) = \sqrt{3}\ or\ \frac{1}{\sqrt{3}}$. These (for $-\pi/2 < x < \pi/2$)occur when $\sin(x) = \sqrt{3}/2$ and $\cos(x) = 1/2$, or vice versa. That is $x = \pi/6 + n\pi$ or $\pi/3 + n\pi$, for all integers $n$.