TD Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

April 25, 2007 LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

SENIOR INDIVIDUAL COMPETITION

Grades 11 and 12

Name: _____

E-Mail: _____

School & Grade: _____

Telephone: _____

Question #	*Your Answer*	For Markers Use only
1		/3
2		/3
3		/3
4		/3
5		/3
6		/3
7		/3
8		/3
9		/3
10		/3
11		/4
12		/4
13		/4
14		/4
15		/4
	Number of Unanswered Questions	x 1
		/50

For Markers use (full solution):

Question #	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Full Solution Total	/50

Instructions for full solution questions:

- Place your solutions to these questions in this answer booklet.
- If you require additional space, use the back of the page but leave a note indicating this to the marker.

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Senior Multiple Choice Problems:

1. Which of these is an equation for the line passing through (2,5) and (4,11)?

   (a) $y = x - 3$ (b) $y = 2x - 1$ (c) $y = 4x - 3$ (d) $y = x^2 - 3x + 7$ (e) $y = 3x - 1$

2. For any positive integer n, let $n! = n(n-1)(n-2) \ldots (2)(1)$. What is the last digit of $1! + 2! + 3! + \ldots + 50!$?
   (a) 0 (b) 3 (c) 4 (d) 5 (e) 9

3 Michelle was asked to add 12 to a certain number, and then divide the result by 4. Instead, she first added 4 and then divided by 12. She ended up with 5 as an answer. If she followed instructions correctly, what would her result have been?
   (a) 5 (b) 17 (c) 20 (d) 56 (e) 60

4. The parabola $y = x^2 - 10x + k$ will have its vertex located on the x-axis when $k = \ldots$

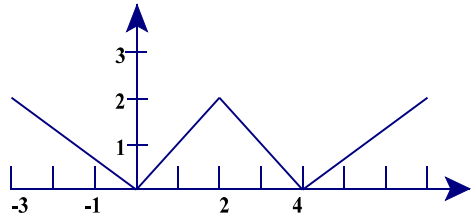
   (a) -100 (b) -25 (c) 0 (d) 25 (e) 100

5. A company estimates that the number of copies of their new DVD that they will sell, in thousands, is given by

$$\frac{3000}{2p + a}$$

   where p is the price in dollars, and a is a constant. If their estimates suggest that 100 000 copies will be sold if they charge \$10 per DVD, what would their estimate be if they charged \$20?
   (a) 10 000 (b) 60 000 (c) 85 000 (d) 100 000 (e) 150 000

6, If there are ten students in a club, then the number of ways to choose a president, vice-president and treasurer (assuming that nobody gets more than one job) is
   (a) 3 (b) 27 (c) 120 (d) 720 (e) 1000



7. Which function is depicted below?

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(a) $|x-2|+2$ (b) $|x+2|+2$ (c) $||x+2|+2|$ (d) $|x+2|-2$ (e) $||x-2|-2|$

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8. On a sheet of paper are listed the following four statements.
This page contains exactly one false statement.
This page contains exactly two false statements.
This page contains exactly three false statements.
This page contains exactly four false statements.

How many of the statements are actually false?
(a) none (b) 1 (c) 2 (d) 3 (e) all of them

9. A sequence is constructed so that the difference between consecutive terms is a constant.
If the first four terms are $x, y, 3x + y$, and $x + 2y + 2$, find $y-x$.

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

10. How many solutions (x,y), with $0 \le x \le \pi, 0 \le y \le \pi$, are there to $\sin(x + y) = \sin (x - y) = 0$?

(a) 0 (b) 2 (c) 4 (d) 5 (e) infinitely many

11. If a positive number x satisfies $x^2 + \frac{1}{x^2} = 4$, find $x + 1/x$.

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{6}$ (e) 6

12. Find the smallest positive integer n such that $2^{1/7}2^{3/7}\ldots 2^{(2n+1)/7} > 1000$.

(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

13. How many real solutions does the following equation have? $x^6 - 3x^4 + 3x^2 - 1 = 0$

(a) none (b) 1 (c) 2 (d) 6 (e) infinitely many

14. If we make the statement that $|x^2 - 9| < a$ whenever $|x - 3| < .1$, then what is the smallest value for a that will make this true?

(a) .01 (b) .1 (c) .2 (d) .21 (e) .61

15. If we let $a * b = \frac{a+b}{ab}$, then $6 * (3 * 3) =$?

(a) 1/2 (b) 2/3 (c) 5/3 (d) 7/3 (e) 54

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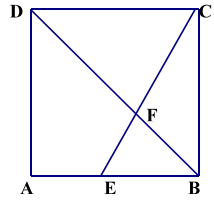
Senior Proof Questions:

1. In a square ABCD, with each side having length 1, let E be the midpoint of AB, and F the intersection of DB and CE. Find the area of the triangle BEF.



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2. Simplify the expression $(\log_2 3)\,(\log_3 4)\,(\log_4 5)\,\ldots\,(\log_{63} 64)$.

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3. For any number x, let $[x]$ denote the largest integer less than or equal to x, and $\{x\} = x - [x]$. (For example, $\{5.7\} = 5.7 - 5 = 0.7$.)

If $y = \dfrac{\left\{\sqrt{3}\right\} - 2\left\{\sqrt{2}\right\}}{\left\{\sqrt{3}\right\}^2 - 2\left\{\sqrt{2}\right\}^2}$, find $[y]$.

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4. Let n be a positive integer. Show that if $2^n - 1$ is a prime number, then n is a prime number.

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5. Find all x such that $\sin^2(x) - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\sin(x)\cos(x) + \cos^2(x) = 0$.