Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

April 25, 2007 LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

SENIOR TEAM COMPETITION

Grades 11 and 12

School Name: ______________________________

Team Members #1 ______________________________ PH: ______________

#2 ______________________________ PH: ______________

#3 ______________________________ PH: ______________

Question #	**For Markers Use only**
1	**/10**
2	**/10**
3	**/10**
4	**/10**
5	**/10**
6	**/10**
7	**/10**
8	**/10**
9	**/10**
10	**/10**
GRAND TOTAL	**/100**

Instructions for full solution questions:

- Place your solutions to these questions in this answer booklet.
- If you require additional space, use the back of the page but leave a note indicating this to the marker.

Team: ____________________________

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1. Many numbers can be written using precisely four 4's and other mathematical symbols such as $+$, $-$, $\times$, $\div$, $\sqrt{\ }$, $!$, and brackets. For instance,

$0 = 44 - 44$
$1 = 44 \div 44$
$2 = (4 \times 4) \div (4 + 4)$.

Find such expressions for the numbers 3, 4, . . . , 12.

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2. Let n be a positive integer. What remainder do you get if you divide $1^n + 2^n + 3^n + 4^n$ by 4?

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3. Five students sit around a round table. Each one thinks of a number and whispers it in the ear of the student to his left and the student to his right. Each student then calculates the average of the two numbers he has heard and announces it. Proceeding clockwise around the circle, the numbers announced are 1, 2, 3, 4 and 5. What number did the person who announced the number 4 think of?

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4. An equilateral triangle has the same perimeter as a regular hexagon (that is, a hexagon with all sides and all angles equal). If the area of the triangle is 10, what is the area of the hexagon?

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5. How many points of intersection do the curves $y = 5\log x$ and $y = \log(5x)$ have?

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6. For each positive integer n, let $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}$.

Show that $a_n < a_{n+1}$ for all n, and that each $a_n < 1$.

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7. Find the sum of all four-digit palindromic numbers. A number is said to be palindromic if it reads the same forward and backward, like 2332.

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8. Find $\sum_{n=1}^{2007}\left(n\cos^2\left(n\pi/2\right)-n\sin^2\left(n\pi/2\right)\right)$.

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9. If we select 9 points in a unit cube, show that two of those points must be within $\sqrt{3}/2$ units of each other. Also show that no number smaller than $\sqrt{3}/2$ will suffice.

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10. Let P be a point inside a circle. Draw every possible line segment from P to the circle, and take the midpoint of each line segment. Show that these midpoints form a circle. How does its radius compare to that of the original circle?