Math 4213 (Differential Geometry) Course Outline

Winter 2024

Instructor: A. J. Dean email: andrew.j.dean@lakeheadu.ca

Office Hours: Wednesdays 10:00am-12:00 noon (RB 2009).

References:
Elementary Differential Geometry, by A.N. Pressley, Springer.

Differential Geometry: Curves–Surfaces–Manifolds, 2nd ed., By W. Kühnel, American Mathematical Society.

Calculus on Manifolds, by Michael Spivak, Perseus Books.

Learner Outcomes: After successfully completing this course, the student will be familiar with the basic concepts, techniques, and computations of the differential geometry of curves and surfaces. They will be proficient in computations with parametrisations of curves, including position, velocity, and acceleration; curvature; evolutes and involutes; and natural equations. They will be familiar with basic concepts and properties of plane curves, including rotation index, the isoperimetic inequality, curvature, convexity, and the four-vertex theorem. They will be familiar with basic properties of curves in space, including: curvature, torsion, and the Frenet frame; the osculating plane and osculating sphere; natural equations; and indicatrices and total curvature, knots and links. They will be proficient in visualising and performing computations with parametrised surfaces, tangent planes and regular surfaces, and changes of coordinates. They will be familiar with tangent spaces and the normal vector, and the concept of orientable surfaces. They will understand, and be able to perform computations with, the first fundamental form, the Gauss map, and the second fundamental form. They will understand the definitions of, and be able to perform computations with, normal and principal curvatures, gaussian and mean curvatures, ruled surfaces and minimal surfaces. They will become proficient in the use of tensor notation, and familiar with Gauss' equations, and the Christoffel symbols. They will be familiar with the Codazzi equations and the Theorema Egregium, the fundamental theorem of surface theory, curvatures and torsions on surfaces, geodesics, and geodesic coordinates. Time permitting, they will also become familiar with the Gauss-Bonnet theorem and its applications, and intrinsic geometry. The student will also become proficient in presenting mathematical proofs in differential geometry at a blackboard in a clear and effective manor.

Tentative Schedule:

Week 1. Parametrisations; position, velocity, and acceleration; curvature; evolutes and involutes; and natural equations.

Week 2. Basic properties of plane curves; rotation index; isoperimetic inequality; curvature, convexity, and the four-vertex theorem.

Week 3. Definitions, examples, and differentiation of curves in space; curvature, torsion, and the Frenet frame; osculating plane and osculating sphere; natural equations.

Week 4. Basic properties of curves in space, indicatrices and total curvature, knots and links.

Week 5. Parametrised surfaces, tangent planes and regular surfaces, change of coordinates.

Week 6. The tangent space and the normal vector, orientable surfaces.

Week 7. The first fundamental form, the Gauss map, the second fundamental form.

Week 8. Normal and principal curvatures, gaussian and mean curvatures, ruled surfaces and minimal surfaces.

Week 9. Tensor notation, Gauss' equations, and the Christoffel symbols.

Week 10. Codazzi equations and the Theorema Egregium, the fundamental theorem of surface theory.

Week 11. Curvatures and torsions on surfaces, geodesics, geodesic coordinates.

Week 12. The Gauss-Bonnet theorem and applications, intrinsic geometry.

Grading Scheme: There will be eleven homework assignments. These will count for 10% each, with the lowest mark being dropped. Solutions will be presented weekly, and the presentations will form part of the grades.

Drop Date: The final date to withdraw from this course without academic penalty is Friday March 8.

Academic Dishonesty: All cases of academic dishonesty will be dealt with according to the university's Academic Integrity Code.

Accommodations: Lakehead University is committed to achieving full accessibility for persons with disabilities. Part of this commitment includes arranging academic accommodations for students with disabilities to ensure they have an equitable opportunity to participate in all of their academic activities. if you think you may need accommodations, you are strongly encouraged to contact Student Accessibility Services (SAS) and register as early as possible. For more information, please visit: http://studentaccessibility.lakeheadu.ca