TD Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

April 25, 2007 LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

JUNIOR TEAM COMPETITION

Grades 9 and 10

School Name: _____

Team Members #1 _____ PH: _____

#2 _____ PH: _____

#3 _____ PH: _____

Question #	For Markers Use only
1	**/10**
2	**/10**
3	**/10**
4	**/10**
5	**/10**
6	**/10**
7	**/10**
8	**/10**
9	**/10**
10	**/10**
GRAND TOTAL	**/100**

Instructions for full solution questions:

- Place your solutions to these questions in this answer booklet.
- If you require additional space, use the back of the page but leave a note indicating this to the marker.

Team: _____

School: _____

1. Many numbers can be written using precisely four 4's and other mathematical symbols such as $+, -, \times, \div, \sqrt{\ }, !$ and brackets. For instance,

$0 = 44 - 44$

$1 = 44 \div 44$

$2 = (4 \times 4) \div (4 + 4)$.

Find such expressions for the numbers 3, 4, . . . , 12.

Team: _____

School: _____

2. A man wants to give away one million dollars, but he wants each recipient to get an amount which is a power of 7 (e.g. $7^0 = 1$, $7^1 = 7$, $7^2 = 49$ etc.) Furthermore, for any given amount, he doesn't want more than six people to get that amount. How should he give his money away?

Team: _____

School: _____

3. Two trains, A and B are 480 km apart on a straight track. Train A travels towards Train B at 100 km/h and Train B travels towards Train A at 60 km/h. A fly begins at Train A, and starts flying towards Train B at 140 km/h. Whenever the fly reaches a train, it will turn around and start flying towards the other train at 140 km/h. When the two trains collide, how far will the fly have travelled?

Team: _____

School: _____

4. Let n be a positive integer. What remainder do you get if you divide $1^n + 2^n + 3^n + 4^n$ by 4?

Team: _____

School: _____

5. We add a two-digit number to another two-digit number, and the sum is a three digit number. If we have used each of the digits 0, 1, 2, 3, 4, 5, and 6, what is the last digit in the three-digit number?

That is,
$$\begin{array}{r} AB \\ +CD \\ \hline EFG \end{array}$$
Find G.

Team: _____

School: _____

6. Five students sit around a round table. Each one thinks of a number and whispers it in the ear of the student to his left and the student to his right. Each student then calculates the average of the two numbers he has heard and announces it. Proceeding clockwise around the circle, the numbers announced are 1,2,3,4 and 5. What number did the person who announced the number 4 think of?

Team: _____

School: _____

7. Anna, Bob, Carol and Duke need to get across a bridge at night. The bridge is very unsteady and cannot take the weight of more than two of them at once. Also, they have only one flashlight, and nobody will cross the bridge in the darkness. Each of them walks at a different speed, and if two are walking together they must walk at the speed of the slower person. Remember that each time they walk across, somebody must bring the flashlight back. If Anna takes 1 minute to walk across, Bob takes 2 minutes, Carol takes 5 minutes, and Duke takes 10 minutes, explain how to get all of them across the bridge in 17 minutes.

Team: _____

School: _____

8. An equilateral triangle has the same perimeter as a regular hexagon (that is a hexagon with all sides and all angles equal). If the area of the triangle is 10, what is the area of the hexagon?

Team: _____

School: _____

9. Wanda, Xavier, Yolanda and Zeke are all mathematicians. Each of them is either a pure mathematician or an applied mathematician. Pure mathematicians always tell the truth. Applied mathematicians always lie. Also, each of them is either well or unwell. Mathematicians who are well believe that correct things are true and incorrect things are false. Mathematicians who are unwell believe that correct things are false and incorrect things are true. They make the following statements:

Wanda: I am unwell.
Xavier: I am a pure mathematician.
Yolanda: I am an applied mathematician.
Zeke: I am well.
Wanda: Yolanda is a pure mathematician.
Xavier: Zeke is unwell.
Yolanda: Xavier is an applied mathematician.
Zeke: Yolanda is well.

Decide if each of the mathematicians is pure or applied, and well or unwell.

Team: _____

School: _____

10. Let P be a point inside a circle. Draw every possible line segment from P to the circle, and take the midpoint of each line segment. Show that these midpoints form a circle. How does its radius compare to that of the original circle?