JUNIOR INDIVIDUAL COMPETITION - SOLUTIONS
Grades 9 and 10

Multiple Choice:

1. The perimeter is 6 times the width, so the width is 15, and the length is 30.
The answer is (b).

2. If she started with x, then $(x + 4)/12 = 5$, so $x = 56$. Thus, $(x + 12)/4$ gives us (b) 17.

3. As $n!$ is a multiple of 10 for all $n \geq 5$, we need only calculate $1! + 2! + 3! + 4! = 33$, so the last digit is (b) 3.

4. Notice that $E = 420 + 421 + A-845 = A-4$, $D = 847 + A-422 -423 =A+2$, $C = 848 + D -424-425 = D -1 = A + 1$, and $B = 849 + C - 426- 427 = C - 4 = A - 3$, so the smallest is (e).

5. This is $P(10,3) = 720$, so (d)

6. The answer is $3(8)/(2/3) = 36$, so (d)

7. The ratio is always $2\pi:1$ The answer is (e).

8. We have $3000/(2(10) + a) = 100$, so $a = 10$, and therefore $3000/(2(20) + 10) = 60$, so the answer is (b) 60 000

9. A standard calculation gives (e) $y = 3x -1$.

10. The statements all contradict each other, so at most one of them can be true. If all of them were false, there would be four false statements, so the last statement would in fact be true, which is impossible. Therefore, the answer must be (d) 3.

11. Notice that $(x + 1/x)^2 = x^2 + 1/x^2 +2 = 6$, so since $x + 1/x$ must be positive, it is (d) $\sqrt{6}$

12. If $n = k^2$, then the next perfect square is $(k + 1)^2 = k^2 + 2k + 1 = n + 2\ \sqrt{n} + 1$, so the answer is (a).

13. This is $(x^2 - 1)^3 = 0$. The solutions are ± 1, so the answer is (c) 2.

14. We want $2^{(1 + 3 + 5 + ... + (2n + 1))/7} > 1000$. Notice that $2^{10} = 1024$, so we will look for a number close to 70 in the exponent. Now, when $n = 7$, we get $2^{64/7}$ which is just over $2^9 = 512$ and therefore less than 1000, but when $n=8$, we get $2^{81/7}>2^{10}$, so the answer must be (c) 8.

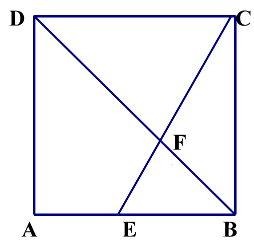
15. We have $3*3=(3+3)/(3(3))=2/3$, so $6*(2/3)=(6+(2/3))/(6(2/3))=5/3$, and the answer is (c).

Key:

1. b
2. b
3. b
4. e
5. d
6. d
7. e
8. b
9. e
10. d
11. d
12. a
13. c
14. c
15. c

Proof Questions:

1. The last element in the 18th set is the total number of elements in the first 18 sets, which is 1 + 2 + . . . +18 = 171. therefore the 19th set is $\{172, 173, \ldots, 190\}$ (as it contains 19 elements), so the sum is 3439.



2. Drop a perpendicular from F to AB, and let us say that it meets AB at G. Draw a perpendicular from F to BC and say it meets BC at H. If we let the length of FG (and FH) be x, then we have divided the triangle EBC into square with side length x and two right triangles, one with legs x and 1 -x, the other with legs x and ½ -x. Since the area of EBC is ¼, we get $1/4 = x^2 + (1/2)x(1-x) + (1/2)(x)(1/2 - x)$, so, $3/4\,x = 1/4$, and $x = 1/3$.

Therefore, triangle BCF has base ½ and height ⅓, so its area is 1/12.

3. Bring the goat across, and leave it. Return, and bring the wolf across. Leave the wolf and bring back the goat. Drop off the goat, and bring the cabbage across. Leave the cabbage, and return. Bring the goat across.

4. We have $y = \dfrac{\sqrt{3} - 1 - 2\left(\sqrt{2} - 1\right)}{\left(\sqrt{3} - 1\right)^2 - 2\left(\sqrt{2} - 1\right)^2} = \dfrac{\sqrt{3} - 2\sqrt{2} + 1}{-2\sqrt{3} + 4\sqrt{2} - 2} = -0.5.$

   Thus [y] = [-0.5] = -1

5. If n = 1, then 2^n -1 = 1, which is not prime, so we will assume that $n > 1$, and therefore, that n is composite. Let $n = ab$, with $a > 1$ and $b > 1$.

   Then $2^{ab} - 1 = \left(2^a - 1\right)\left(1 + 2^a + 2^{2a} + \ldots + 2^{a(b-1)}\right)$, and therefore we have a composite number, as each of the factors is larger than 1. This is a contradiction.