TD Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

April 25, 2007 LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

JUNIOR INDIVIDUAL COMPETITION

Grades 9 and 10

Name: _____

E-Mail: _____

School & Grade: _____

Telephone: _____

Question #	*Your Answer*	For Markers Use only
1		**/3**
2		**/3**
3		**/3**
4		**/3**
5		**/3**
6		**/3**
7		**/3**
8		**/3**
9		**/3**
10		**/3**
11		**/4**
12		**/4**
13		**/4**
14		**/4**
15		**/4**
	Number of Unanswered Questions	**x 1**
		/50

For Markers use (full solution):

Question #	Mark
1	**/10**
2	**/10**
3	**/10**
4	**/10**
5	**/10**
Full Solution Total	**/50**

Instructions for full solution questions:

- Place your solutions to these questions in this answer booklet.
- If you require additional space, use the back of the page but leave a note indicating this to the marker.

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Multiple Choice Questions:

1. A rectangle is twice as long as it is wide. If the perimeter is 90 cm, what is the area, in square cm?

   (a) 225 (b) 450 (c) 506.25 (d) 900 (e) 8100

2. Michelle was asked to add 12 to a certain number, and then divide the result by 4. Instead, she first added 4 and then divided by 12. She ended up with 5 as an answer. If she followed the instructions correctly, what would her result have been?

   (a) 5 (b) 17 (c) 20 (d) 56 (e) 60

3. For any positive integer n, let $n! = n(n - 1)(n-2)...(2)(1)$. What is the last digit of $1! + 2! + 3! + ... + 50!$?

   (a) 0 (b) 3 (c) 4 (d) 5 (e) 9

4. Let $A = 422 + 423 + ... + 846$, $B = 428 + 429 + ... + 849$, $C = 426 + 427 + \ldots + 848$, $D = 424 + 425 + \ldots + 847$, and $E = 420 + 421 + \ldots + 845$. Which of these numbers is the smallest?

   (a) A (b) B (c) C (d) D (e) E

5. If there are ten students in a club, then the number of ways to choose a president, vice-president and treasurer (assuming that nobody gets more than one job) is

   (a) 3 (b) 27 (c) 120 (d) 720 (e) 1000

6. Three cans of water fill two-thirds of a one-litre jug. How many cans are needed to fill 8 one-litre jugs?

   (a) 16/3 (b) 16 (c) 24 (d) 36 (e) 48

7. If the area of a circle is doubled, then the ratio of the circumference to the radius

   (a) is doubled (b) is multiplied by 4 (c) is multiplied by $\sqrt{2}$ (d) is multiplied by $\sqrt{2\pi}$ (e) remains the same

8. A company estimates that the number of copies of their new DVD that they will sell, in thousands, is given by $\dfrac{3000}{2p + a}$

   where p is the price in dollars, and a is a constant. If their estimates suggest that 100 000 copies will be sold if they charge \$10 per DVD, what would their estimate be if they charged \$20?

   (a) 10 000 (b) 60 000 (c) 85 000 (d) 100 000 (e) 150 000

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9. Which of these is an equation for the line passing through (2,5) and (4,11)?

(a) $y = x - 3$ (b) $y = 2x - 1$ (c) $y = 4x - 3$ (d) $y = x^2 - 3x + 7$ (e) $y = 3x - 1$

10. On a sheet of paper are listed the following four statements.
This page contains exactly one false statement.
This page contains exactly two false statements.
This page contains exactly three false statements.
This page contains exactly four false statements.

How many of the statements are actually false?
(a) none (b) 1 (c) 2 (d) 3 (e) all of them

11. If a positive number x satisfies $x^2 + \frac{1}{x^2} = 4$, find $x + 1/x$.

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{6}$ (e) 6

12. If n is a perfect square, then what is the next perfect square after n? (For example, if $n = 49$, then the next perfect square is 64.)

(a) $n + 2\sqrt{n} + 1$ (b) $n^2 + 2n + 1$ (c) $\sqrt{n^2 + 2n + 1}$ (d) $\sqrt{n^2 + 1}$ (e) none

13. How many real solutions does the following equation have? $x^6 - 3x^4 + 3x^2 - 1 = 0$

(a) none (b) 1 (c) 2 (d) 6 (e) infinitely many

14. Find the smallest positive integer n such that $2^{1/7} 2^{3/7} \dots 2^{(2n+1)/7} > 1000.$

(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

15. If we let $a * b = \frac{a+b}{ab}$, then $6 * (3 * 3) = ?$

(a) 1/2 (b) 2/3 (c) 5/3 (d) 7/3 (e) 54

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1. Consider the sets of integers $\{1\}$, $\{2,3\}$, $\{4,5,6\}$ . . . , with each set having one more element than the previous set. Find the sum of the elements in the 19th set.

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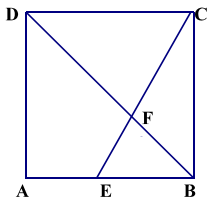
2. In a square ABCD, with each side having length 1, let E be the midpoint of AB and F the intersection of DB and CE. Find the area of the triangle BEF.



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3. A farmer owns a wolf, a goat and a cabbage. He must make it across a river in a boat that can hold only himself and one of his three possessions. If he leaves the wolf alone with the goat, the goat will get eaten. If he leaves the goat alone with the cabbage, the cabbage will get eaten. The farmer must be in the boat for each trip. Explain how he can accomplish this without anything getting eaten.

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4. For any number x, let $[x]$ denote the largest integer less than or equal to x, and $\{x\} = x - [x]$. (For example, {5.7} = 5.7 -5 = 0.7.)

If $y = \dfrac{\{\sqrt{3}\} - 2\{\sqrt{2}\}}{\{\sqrt{3}\}^2 - 2\{\sqrt{2}\}^2}$, Find [y].

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5. Let n be a positive integer. Show that if $2^n - 1$ is a prime number, then n is a prime number. (Recall that a positive integer is said to be prime if it is greater than 1, and if it cannot be written as a product of two positive integers, unless one of them is 1.)