**Junior Team Solutions - TD Canada Trust High School Mathematics Competition 2011**

1. Since $7 \times C$ has units digit 8, we must have $C = 4$. Thus, $7 \times B + 2$ has units digit 9, so $B = 1$. $E$ is the units digit of $7 \times 6$, so $E = 2$. Now, $7 \times A + 4$ has units digit 9, so $A = 5$. Finally, $5614 \times 7 = 39298$, so $D = 3$. Thus, $A = 5, B = 1, C = 4, D = 3, E = 2$.

2. John repeats the following sequence: LRLRLBS, where L and R are ordinary left and right steps, B is a backward left step, and S is a 1 m right step. This is 7 steps, and the distance covered is $5(.5) - .5 + 1 = 3$ metres. Doing this 6 times will take 42 steps and cover 18 m. John then takes an additional L and an additional R to bring him to his destination. Thus, 44 steps are required.

3. The trains approach each other at 60 km/h. Since they start out 60 km apart, this takes 1 hour. If the dragonfly travels 40 km/h, then it will travel 40 km.

4. Solution 1: We have $(x - 1)(y - 1) = xy - (x + y) + 1 = 1$, since $xy = x + y$. But if two integers multiply together to give 1, then they are both 1 or both $-1$. But then $x - 1 = y - 1 = 1$ means that $x = y = 2$, and $x - 1 = y - 1 = -1$ means that $x = y = 0$.

Solution 2: If either $x = 0$ or $y = 0$, then we will have $x + y = 0$, so in fact, $x = y = 0$ (and this case obviously works). Assume neither $x$ nor $y$ is 0. Then dividing by $xy$, we get $(1/y) + (1/x) = 1$. Now, if $x < 0$, then $1/y = 1 - (1/x) > 1$, which is impossible, so $x > 0$ (and similarly $y > 0$). Since we have two terms summing to 1, one must be at least as big as 1/2. But letting either $x = 1$ or $y = 1$ doesn't work, so either $x = 2$ or $y = 2$. But then we see that $x = y = 2$ is the other solution.

5. We want to show that

$$\frac{a+c}{b+d} - \frac{a}{b} > 0.$$

But

$$\frac{a+c}{b+d} - \frac{a}{b} = \frac{(a+c)b - a(b+d)}{b(b+d)} = \frac{bc - ad}{b(b+d)}.$$

Now, the denominator is positive, and since $a/b < c/d$, we get $bc > ad$, and this part is done. Similarly

$$\frac{c}{d} - \frac{a+c}{b+d} = \frac{c(b+d) - (a+c)d}{d(b+d)} = \frac{bc - ad}{b(b+d)}.$$

which gives the second inequality.

6. Notice that the percentage of students liking both shows is 80% of 60%, or 48%. Thus, the fraction of Xena fans who are also Buffy fans is 48/70 (or 24/35, or approximately 68.6%).

7. First, we may assume that the vertices of the triangle lie on the rectangle. If not, then we shrink the rectangle both horizontally and vertically until the vertices lie on it. Doing so does not affect the triangle, but reduces the perimeter of the rectangle, thus making our conclusion stronger. Now, if the triangle is ABC, then to get the perimeter, we travel from A to B, B to C, then C to A, in each case using a straight line. But we can also calculate the perimeter of the rectangle by going from A to B, B to C, C to A, following instead the rectangle. But now we aren't following straight lines anymore (at least, not in all 3 parts). Since the shortest distance between 2 points is a straight line, the perimeter of the triangle is smaller.

8. We must select 3 positions for the 2, 4 and 8. (The order is chosen for us.) There are 10 ways to do it: $248AB, 24A8B, 24AB8$, etc. Of the remaining 2 positions, we have 6 choices for the first digit (anything other than 0, 2, 4 or 8) and 5 for the second (not 0, 2, 4, 8 or the digit we just selected). The total number is $10(6)(5) = 300$.

9. Observe that

$$12^3 = (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + y^3) + 3(x+y)(xy) = 36 + 3(12)(xy).$$

Thus, $xy = (12^3 - 36)/36 = 47$. But also,

$$36 = x^3 + y^3 = (x+y)(x^2 - xy + y^2) = 12(x^2 + y^2 - 47).$$

That is, $x^2 + y^2 = (36/12) + 47 = 50$.

10. Let $r$ be the number of red chameleons, $b$ the number of blue and $g$ the number of green, at any given moment. Let $x = r - b$, $y = b - g$ and $z = g - r$. Let us consider the meeting of a red and a blue. Then $r$ and $b$ each drop by one, but $g$ increases by 2. We see that $x$ doesn't change, $y$ decreases by 3 and $z$ increases by 3. The other possible meetings are similar: one of $\{x, y, z\}$ stays the same, and the other 2 change by $\pm 3$. If all chameleons ever become the same colour, then two of $\{r, b, g\}$ must be 0. In particular, one of $\{x, y, z\}$ will be zero. But if $x$, $y$ and $z$ only ever change by 3 at a time, then the only way we can get to 0 is if one of them is a multiple of 3 to begin with. However, we start with $x = -7$, $y = -10$, $z = 17$, and none of these are multiples of 3. Therefore, it is impossible.