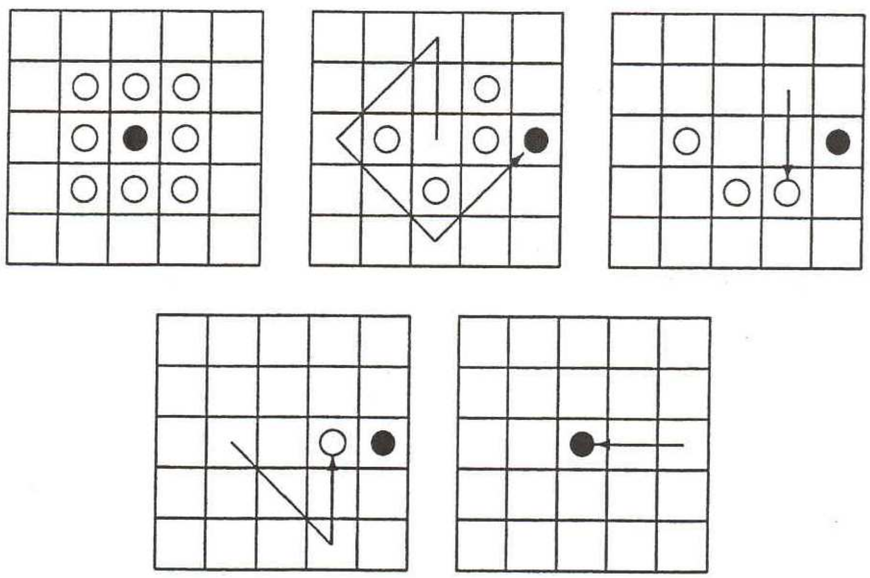
TD Canada Trust Northwestern Ontario High School Mathematics Competition

Junior Team Solutions

1. There are many possible solutions for each triple. Here is one solution for each triple:
1×2^3, $2 \times 2 \times 2$, $7 + (6 \div \sqrt{9!})$, $8 + \sqrt{9} - 3$, $(5 - 2) + 5$.

2. Let x be the number of CDs for Bob, and y for June. We are given $x = y + 5$, and $2x + 7y = 100$. Since $x = y + 5$, we have $2y + 10 + 7y = 100$, i.e., $9y = 90$. So $y = 10$, $x = 15$, and the combined collection has 25 CDs.

3. You can do this in 4 jumps. The trick is to note that you first want to use the middle black counter to remove 4 pieces (in a single jump). The last jump puts the black counter back in the middle.



4. After completing n loops of the spiral (counting 1 as loop zero), we will have a $(2n+1) \times (2n+1)$ grid, with the last entry, $(2n+1)^2$, in the lower left corner. The lower right corner is therefore $(2n+1)^2 - 2n$, and the middle of the rightmost column would be $(2n+1)^2 - 2n - n = 4n^2 + n + 1$. So 1, 6 and 19 correspond to $n = 0$, 1, and 2 respectively. The next three numbers are 40, 69 and 106. [Note: The general solution will be depend on where you start your numbering, so $4(n-1)^2 + (n-1) + 1$ would be fine too.]

5. Since $Z = \{1, 2, \ldots, 9\}$, the average value is 5.

6. The area of the smallest circle is π, so each shaded region has area $\pi/4$. If the middle circle has radius r, then the area of its shaded region is $\pi r^2/4 - \pi/4$, so setting this equal to $\pi/4$, we get $r = \sqrt{2}$. If the radius of the largest circle is R, then the area of its shaded region is $\pi R^2/4 - \pi r^2/4 = \pi R^2/4 - \pi/2$. Setting this equal to $\pi/4$, we get $R = \sqrt{3}$.

7. There are only 12 cards with even numbers to go around, and 13 students, so at least one student will have to get 2 odd numbers. Adding them together gives an even number, and an even number multiplied by anything else is even.

8. If there are n races, then $n(x+y+z) = 10+20+9 = 39$. So, n has to be a divisor of 39, that is, n must be among $1, 3, 13$, or 39. As $x+y+z \geq 3+2+1 = 6$, $n = 1$ or 3. But we are told there are at least two races, so $n = 3$. Now, June got only 9 points, so $z \leq 3$. But if $z = 3$, then June lost all of the races, so Bob's score must be at least $4+4+4$, and it is not. Therefore, $z \leq 2$. But June cannot have three different placings, as then her total would be $x+y+z = 13$. If $z = 2$, this means her scores are $2+2+5$ or $3+3+3$. Then we have $x = 6$, $y = 5$, $z = 2$, or $x = 8$, $y = 3$, $z = 2$ and either way, a score of 20 is impossible. So $z = 1$. Again, June must have at least two matching scores, so she has $3+3+3$, $2+2+5$, $1+1+7$ or $1+4+4$. The first and third cases make a score of 20 impossible, and the second doesn't work, as $1+2+5 \neq 13$. So $x = 8$, $y = 4$, $z = 1$. Since Michelle came second in one race, June came second in the other two races, so the answer is June.

9. Multiply the equation $3(2^x) = y^2 - 4$ by 2^{2006}. Then we get $3(2^{x+2006}) = (2^{1003}y)^2 - 2^{2008}$. That is, if (a, b) is a solution to our original equation, then $(a+2006, 2^{1003}b)$ is a solution to our desired equation. Therefore, $(2008, 4(2^{1003}))$, $(2011, 10(2^{1003}))$ and $(2012, 14(2^{1003}))$ are the answers.

10. Each term describes the preceding term in words. The first entry is "1 one", so we write "11". This is "2 ones", so we write "21". Then "1 two and 1 one", so "1211", and so on. The next three terms are 31131211131221, 13211311123113112211, 11131221133112132113212221.