TD Canada Trust

Lakehead University
Department of Mathematical Sciences

LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION
April 30, 2008.

JUNIOR TEAM COMPETITION
Grades 9 and 10

School Name: ______________________________

Team Members: #1 ______________________________ PH: ____________________

#2 ______________________________ PH: ____________________

#3 ______________________________ PH: ____________________

Question #	For Markers Use only
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
GRAND TOTAL	/100

Team Members: ____________________

School: ____________________

1. Given any three numbers from 1 through 9, you can write an expression for 8 using the three numbers, the symbols $\times, \div, +, -, !, \sqrt{}$, and brackets. (Here, $n! = 1 \cdot 2 \cdot 3 \cdots n$ is the product of the integers from 1 to n.) For example, consider the three numbers $3, 6$ and 7. Then

$$7 + 6 \div (3!) = 8.$$

(You must use 3 exactly once, because it appears exactly once in the list.)

Find expressions for 8 using the following triples: $(1, 2, 3), (2, 2, 2), (6, 7, 9), (3, 8, 9), (2, 5, 5)$.

Team Members: ____________________

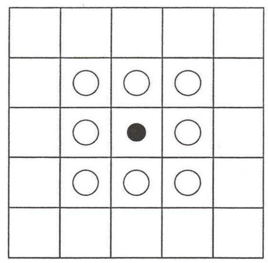
School: ____________________

2. Bob and June each have some CDs. Bob's CDs have 2 songs each, and June's have 7 songs each. Bob has 5 more CDs than June. The total number of songs on all of their CDs combined is 100, and none of the songs appear more than once. How many CDs do they have together?

Team Members: ____________________

School: ____________________

3. On a 5×5 chessboard, place one black counter in the middle of the board, and put eight white counters in the spaces surrounding the black counter. Each counter may jump over an adjacent counter (horizontally, vertically or diagonally) provided that the square immediately beyond is vacant. The jumping counter will land on that square, while the jumped counter is removed (note that the colour doesn't mater; a white counter jumped over a white counter will remove the white counter). Remove all eight white counters this way, while returning the black counter to its initial position. Use the smallest possible number of jumps, where a sequence of consecutive jumps by the same counter counts as a single jump. (In your solution, draw the board after each move. As long as your solution uses the smallest possible number of moves, you don't need to prove this.)



Team Members: ____________________

School: ____________________

4. In the number spiral below, give the next three numbers to the right of 11, 2, 1, 6, 19.

13	14	15	16	17			
12	3	4	5	18			
11	2	1	6	19	?	?	?
10	9	8	7	20			
		...	22	21			

Now give a general formula for the numbers in the sequence $1, 6, 19, \ldots$ that appear as part of the middle row, as shown.

Team Members: ____________________

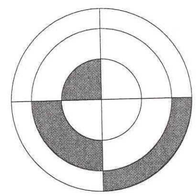
School: ____________________

5. Pick any number x between 10 and 99. Subtract the sum of the two digits in x from x itself, to obtain a number y. Divide the number y by 9 to obtain a result z. Let Z be the set of possible z's obtained from the above algorithm. What is the average of the values in Z?

Team Members: ____________________

School: ____________________

6. In the diagram below, two perpendicular lines meet at the centre of three circles (all of which have the same centre). Each shaded region has the same area. If the radius of the smallest circle is 1, find the radii of the other two circles. (Note: The diagram is not to scale.)



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School: ______________________

7. A half deck of cards consists of 26 cards, each labeled with an integer 1 through 13. There are two cards labeled with a 1, two cards labeled with a 2, and so on. In a particular math class, there are 13 students. The teacher deals each student two cards randomly from the deck. The students are then asked to add their two numbers together. Then, all 13 of the added numbers are multiplied together. The students will have to do their math homework if the number formed by multiplying all the numbers together is even, and they will not have to do their homework if the number is odd. Explain why the students will always have to do their homework.

Team Members: ______________________

School: ______________________

8. Three students (Bob, June and Michelle) are competing in a series of races. In each race, the student coming first gets x points, the student coming second gets y points and the student coming third gets z points, where x, y and z are positive integers. No ties will occur, and $x > y > z$. If Bob got 10 points, Michelle got 20 points, and June got 9 points, and Michelle came second in the second race, who came second in the first race? Be sure to explain why your answer is the only possible solution. Note that you need to first figure out the number of races. As well, Bob, Michelle, and June only receive their points after all the races are finished.

Team Members: ______________________

School: ______________________

9. Consider the equation $3(2^x) = y^2 - 4$. There are three pairs of positive integers that solve this equation, namely $(2, 4)$, $(5, 10)$ and $(6, 14)$. The equation $3(2^x) = y^2 - 16$ also has three pairs of positive integers that solve it, one of which is $(4, 8)$. Find three pairs of positive integers that solve the system $3(2^x) = y^2 - 2^{2008}$.

Team Members: ____________________

School: ____________________

10. What are the next three terms in the following sequence that starts:

$$1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, \ldots$$

Explain your rule.