TD Canada Trust Northwestern Ontario High School Mathematics Competition

2009 Junior Team Solutions

1. As both terms are positive, this is the same as asking if the square of one is larger than the square of the other. But $(1/\sqrt{2}-1/\sqrt{6})^2 = 1/2+1/6-1/\sqrt{3}$, so we would like to know if this is larger than $9/100$. That is, we want to know if $1/2+1/6-9/100$ is greater than $1/\sqrt{3}$ or, equivalently, if $(1/2+1/6-9/100)^2$ is greater than $1/3$. A quick calculation reveals that it is, so $1/\sqrt{2} - 1/\sqrt{6} > 3/10$.

2. This is equivalent to $x^{256} = 2^{256}$, or $(x/2)^{256} = 1$. That is, $x/2 = \pm 1$, hence $x = \pm 2$. The sum of the squares of the solutions is $2^2 + (-2)^2 = 8$.

3. Expanding gives $(-1)^{2009} = -1$.

4. Let the number be $1abcde$. Then $abcde1 = 3(1abcde)$. To make the units digits work out, we must have $e = 7$. But then $abcd71 = 3(1abcd7)$. To make the tens digits work out, $d = 5$. Similarly, then, $c = 8$, $b = 2$, $a = 4$, so the number is 142857.

5. Every multiple of 5 contributes one 5, but every multiple of 25 contributes a second 5, every multiple of 125 contributes a third 5, and every multiple of 625 contributes a fourth 5. Counting the multiples, we get that $k = 200 + 40 + 8 + 1 = 249$.

6. Taking the 30-th power of each side, we get $10^3 = 1000$ and $2^{10} = 1024$. As the latter is larger, $2^{1/3}$ is larger.

7. If $a > \sqrt{2}$, then there is no intersection. If $a = \sqrt{2}$, then there are two points of intersection, $(1/\sqrt{2}, \pm 1/\sqrt{2})$. If $a = 1$, then we get three intersections; the origin, and one more on each of the lines $y = x$ and $y = -x$. For all other positive values of a, the circle meets each line twice, so there are 4 solutions. There is never precisely one solution.

8. Let a be the number of apples at the beginning. After encountering the first guard, he has $(a/2) - 2$. After the second guard, he has $((a/2)-2)/2-2 = (a/4)-3$. After the third guard, he has $((a/4)-3)/2-2 = (a/8) - (7/2) = 1$. Thus, $a = 36$.

9. If k holes are occupied, then since they all contain different positive integers, the sum must be no less than $1 + 2 + \cdots + k = k(k + 1)/2$. As this cannot exceed 100, $k \leq 13$. The numbers $1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14$ work, so the answer is indeed 13.

10. B is shorter than everyone else in his column, which will include one person in the row containing A. But A is at least as tall as everyone in that row, so A is at least as tall as B. As A and B are not the same person, and all have different heights, A is taller than B.