2006 Thunder Bay High School Mathematics Competition

Junior Team Answers

For complete solutions to any of the problems or for discussions of any of the problems, email ryanholm72@hotmail.com.

Note to markers: For the full solution questions students are expected to provide much more detail than I have here to receive full marks; this is just a reference sheet.

Note to students: The solutions provided here are only *sketch solutions*. Steps have been omitted and the solutions are meant to guide any student who has made a valiant attempt at solving the problem.

Full Solution (Answers and sketch solutions):

1. 20. We can group the subtraction as follows: $(2+4+...+40)-(1+3+...+39)=(2-1)+(4-3)+...(40-39)=1+1+...1=20$.

2. 6, 15. Drawing a picture of each situation, it is clear that whenever you have drawn n distinct lines, it is possible to draw another line intersecting each of these exactly once and it is not possible to get more intersections as each line can intersect each other line at most once. Hence, for four lines there are a maximum of 1 + 2 + 3 = 6 intersections, and 1 + 2 + 3 + 4 + 5 = 15 for the case of six distinct lines.

3. π. Suppose the radius of the circle is R units. Then the radius of the new circle is R + 1 units. Hence, the circumference of the new circle is $2\pi(R+1)$ units and the diameter of the new circle is $2(R+1)$ units so that the ratio is π.

4. No. Suppose he were to put 1 in the first bag, 2 in the second bag, 3 in the third bag, …, 10 in the tenth bag. Then he would need a total of 1 + 2 + 3 +…+ 10 = 45 hamburgers.

5. (ii), (iv) are necessarily true. Since it isn't true that all shirts in the store are on sale, we must have that some shirt in the store isn't on sale. Equivalently, this is the same as saying that not all shirts in this store are on sale. It is easy to come up with examples to show that (i) and (iii) can be false. For example, suppose the store has 4 shirts, 3 of which are on sale and 1 of which isn't. Then the given statement is clearly false and neither (i) or (iii) is true.

6. $\frac{N^3}{M^2}$. Since M men working M^2 hours produce M articles, one man working M^2 hours produces one article. Therefore, one man produces $\frac{1}{M^2}$ of one article in one hour, so N men working N hours a day for each of N days will produce $N \times N \times N \times \frac{1}{M^2} = \frac{N^3}{M^2}$ articles.

7. Ryan, 40 sec. When Ryan begins to walk to class James has already walked $2 \times 120 = 240$ metres. Hence, he has to traverse 516 metres to Ryan's 756 metres. This will take James 516/2=208 seconds and Ryan 756/4.5=168 seconds. Hence, Ryan will arrive 208 – 168 = 40 seconds before James.

8. 3, 3, 8. Listing all the twelve decompositions of 72 into three factors and noting the sum of the factors, there is only one sum occurring twice, namely $14 = 2 \times 6 \times 6 = 3 \times 3 \times 8$ (and this is why the guest had noticed that originally the problem was indeterminate). Since the two youngest are twins, the desired decomposition is the latter.

9. 72 m. Drawing a picture helps. It is then easy to see that each of the 6 outer faces of the cube contributes 8 metres to the total surface area, each of the 6 holes in the faces contributes 4 metres to the total surface area, and that this constitutes the total surface area. Hence, the surface area is $6\times 8+6\times 4=72$ metres.

10. 3, 2, $5.11. We must have that _679_ is divisible by 72. So it must be divisible by both 8 and 9. If it is divisible by 8, the number 79_ must be divisible by 8 (since 1000 is divisible by 8) and so 79_ must be 792. If _6792 is divisible by 9, the sum of its digits must be divisible by 9 and so the first faded digit must be 3. Finally, $\$367.92\div 72=\$5.11$.

11. Suppose Sammy has x stamps of which y sevenths are in the second book. Then we have that $\frac{2x}{10}+\frac{yx}{7}+303=x$. Rearranging this gives $x=\frac{3\times 5\times 7\times 101}{28-5y}$. Since x and y must be positive integers, the only possible solution is $y=5, x=3535$.

12. 15, 20, 25. Let a,b,c denote the sides of the triangle with the latter being the hypotenuse. We are given

(1) $a+b+c=60$

(2) $a^2+b^2=c^2$

(3) $\frac{ab}{2}=\frac{12c}{2}$,

(1) following from the perimeter, (2) following since the triangle is a right triangle, and (3) following from calculating the area of the triangle in two different ways. We have the identity $(a+b)^2=a^2+b^2+2ab$ and substituting (1), (2), (3) into this identity gives us $(60-c)^2=c^2+24c$. Solving this gives $c=25$ from which the result follows.