2005 Thunder Bay High School Mathematics Competition

Junior Team Answers

For complete solutions to any of the problems or for discussions of any of the problems, email ryanholm72@hotmail.com.

Note to markers: For the full solution questions students are expected to provide much more detail than I have here to receive full marks; this is just a reference sheet.

Note to students: The solutions provided here are only *sketch solutions*. Steps have been omitted and the solutions are meant to guide any student who has made a valiant attempt at solving the problem.

Full Solution (Answers and sketch solutions):

1. 50. Let n be the number of jellybeans originally in the jar. Since Doyle ate 20% of the jellybeans each day, 80% of the jellybeans are left at the end of each day. So (0.8)[(0.8)n] = 32 and so n = 50. This problem can also be solved by working backwards.

2. 15. Let x be the original number. Then $\frac{x-9}{3} = 43$, which gives that x = 138. So the correct answer is $\frac{138-3}{9} = 15$.

3. Just 60. You can figure this out through trial and error, or notice that since the lowest common multiple of 2, 3, 4, and 5, is 60, then any number that is divisible by 2, 3, 4, and 5 must also be divisible by 60. The only number between 1 and 100 divisible by 60 is 60 itself.

4. (c) is necessarily true because I tells us that there exists students who are not honest and II tells us that such students cannot be math club members. (a) and (b) may not be true because each requires that the set of all math club members be nonempty, which is not required by I or II. Again, I and II would allow the set of all math club members to be a nonempty subset of the set of all students, but neither (d) or (e) permits this and accordingly may be not true.

5. 810-642-9753. The last four digits (GHIJ) are either 9753 or 7531, and the remaining odd digit (either 1 or 9) is A, B, or C. Since A + B + C = 9, the odd digit among A, B, and C must be 1. Thus the sum of the two even digits in ABC is 8. The three digits in DEF are 864, 642, or 420, leaving the pairs 2 and 0, 8 and 0, or 8 and 6, respectively, as the two even digits in ABC. Of those, only the pair 8 and 0 has sum 8, so ABC is 810 and thus the phone number is 810-642-9753.

6. The lowest common multiple of 5, 8, 12, and 18 is 360. So every 360 minutes (6 hours) the buses will all be together at the terminal again. So this will happen twice before 10 pm.

7. 13. Let w, x, y, and z, be the amounts paid by the first, second, third, and fourth boy, respectively. Then
   we get 4 equations in 4 unknowns:
   (I) w + x + y + z = 60
   (II) w = 1/2(x + y + z)
   (III) x = 1/3(w + y + z)
   (IV) y = 1/4(w + x + z)
   Solving this system gives w = 20, x = 15, y = 12, z = 13.

8. 162. Since each match of the tournament eliminates one and only one competitor, no matter how we set up the tournament it will always take exactly 162 games to declare a winner. This is because in each of 162 matches there will be a loser so after 162 matches only 1 person will remain without a loss.

9. The members of 1, 5, 8, and 12 years of experience were on one side and those of 2, 3, 10, and 11 years were on the other. It is easy to check this satisfies the conditions of the problem.

10. $7^{\frac{1}{2}} + 7^{\frac{1}{3}} + 7^{\frac{1}{4}} < 9^{\frac{1}{2}} + 8^{\frac{1}{3}} + 16^{\frac{1}{4}} = 3 + 2 + 2 = 7$. The second statement is not true since in fact $4^{\frac{1}{2}} + 4^{\frac{1}{3}} + 4^{\frac{1}{4}} > 4^{\frac{1}{2}} + 1^{\frac{1}{3}} + 1^{\frac{1}{4}} = 2 + 1 + 1 = 4$.

11. 7. The largest average possible is obtained if 1 is erased – the average is then

$$\frac{2+3+...+n}{n-1} = \frac{\frac{(n+1)n}{2} - 1}{n-1} = \frac{n+2}{2}.$$

The smallest average possible is obtained if n is erased – the average is then

$$\frac{1+2+...+(n-1)}{n-1} = \frac{n(n-1)}{2(n-1)} = \frac{n}{2}.$$

So $\frac{n}{2} \le 35\frac{7}{17} \le \frac{n+2}{2}$, implying that $68\frac{14}{17} \le n \le 70\frac{14}{17}$. Hence, n = 69 or n = 70. Since $35\frac{7}{17}$ is the average of $(n-1)$ integers, $(35\frac{7}{17})(n-1)$ must be an integer and so n = 69. If m is the number erased, then $\frac{\frac{1}{2}(69)(70) - m}{68} = 35\frac{7}{17}$. Solving this for m gives m = 7.

12. Let x be the amount of water present when the pumping begins, y the amount leaking per hour, and z the amount that each person can remove per hour. Then the information in the problem gives us the two equations $x + 3y = 36z$ and $x + 10y = 50z$, since each side of each equation is an expression for the total amount of water removed from the boat. We can solve these two equations for x and y to get that $x = 30z$ and $y = 2z$. We want to find the number n such that $x + 2y = 2nz$. Well, substituting in our expressions for x and y in terms of z gives us that $30z + 4z = 2nz$ and thus $n = 17$.