Junior Individual Solutions - TD Canada Trust High School Mathematics Competition 2011

Multiple Choice Questions.

1. B.
2. D.
3. A.
4. E.
5. B.
6. B.
7. D.
8. A.
9. D.
10. C.
11. E.
12. B.
13. D.
14. C.
15. C.

Full Solution.

1. He uses 1 digit for each of the first 9 pages, 2 for the next 90 and 3 beyond that. (We will not need to go to page 1000.) So for the first 99 pages, he uses $1(9) + 2(90) = 189$ digits; thus, he has $1890 - 189 = 1701$ to go. But $1701/3 = 567$, so he has 567 3-digit pages. Thus, in total, he has $9 + 90 + 567 = 666$.
2. Clearly, to maximize the product, we must make the hundreds digits as large as possible; that is, 5 and 6. (If we choose anything else, the larger number can be no bigger than 654 and the smaller no larger than 465, giving a product no larger than 304110.) So make the numbers $6ab$ and $5cd$. If $a < b$, then swapping a and b increases the product, so $a > b$. Similarly, $c > d$. This reduces us to just a few cases: $643 \cdot 521 = 335003$ (so numbers smaller than 304110 don't work), $642 \cdot 531 = 340902$, $641 \cdot 532 = 341012$, $632 \cdot 541 = 341912$, $631 \cdot 542 = 342002$ and $621 \cdot 543 = 337203$. Thus, the largest product is 342002.
3. $|||x-1|-2|-3|-4 = \pm 5$, so $|||x-1|-2|-3| = 4 \pm 5$. But the left side cannot be negative, so $|||x-1|-2|-3| = 9$. Similarly, $||x-1|-2| = 3 \pm 9$, and since the left side cannot be negative, $||x-1|-2| = 12$. In the same way, $|x-1| = 2 \pm 12$, so we can only have $|x-1| = 14$, Thus, $x - 1 = \pm 14$, and therefore $x = 15$ or -13.