Junior Individual Solutions - TD Canada Trust High School Mathematics Competition 2010

Multiple Choice Questions.

1. C.
2. D.
3. B.
4. D.
5. C.
6. D.
7. B.
8. D.
9. E.
10. A.
11. B.
12. E.
13. A.
14. C.
15. A.
16. D.
17. C.
18. C.
19. A.
20. C.

Full Solution.

1. The largest circle consists of 4 regions, each having the same area as the smallest circle. Thus, the area of the largest circle is 4 times that of the smallest. Since the area is in proportion to the square of the radius, the radius is twice as big. Since the outermost circle has radius 50 m, the innermost one has radius 25 m.
2. (a) If 2010 is written as a sum of two positive integers, then one of them is at least as big as 1005. In particular, its first digit is 1 or 2; thus, it has a digit in common with 2010.
   (b) One example is 13579. (There are many others.) To see why this works, note that if a sum of integers is odd, then one of the integers in the sum is odd, and therefore its last digit is one of the digits in 13579.
3. Let us first count the number of 5-digit numbers with different digits. There are 9 choices for the first digit (anything but 0). There are then 9 choices for the second digit (we can't repeat the first digit, but 0 is allowed). Then disallowing repetition, we get 8 choices for the third digit, 7 for the fourth and 6 for the fifth, a total of $9(9)(8)(7)(6) = 27216$. Let us now solve the same problem, but assume that the digit 3 is not allowed. Then for each digit, the number of choices is reduced by one, hence $8(8)(7)(6)(5) = 13440$. Thus, the answer is $27216 - 13440 = 13776$.