TD Canada Trust

Lakehead UNIVERSITY

Department of Mathematical Sciences

LAKEHEAD UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION
April 30, 2008.

JUNIOR INDIVIDUAL COMPETITION
Grades 9 and 10

Name: _____

School & Grade: _____

Telephone: _____

E-Mail: _____

Question #	*Your Answer*	For Markers Use only
1		/3
2		/3
3		/3
4		/3
5		/3
6		/3
7		/3
8		/3
9		/3
10		/3
11		/4
12		/4
13		/4
14		/4
15		/4
	Number of Unanswered Questions	x 1
		/50

For Markers use (full solution):

Question #	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Full Solution Total	/50

Name: _____

School: _____

Multiple Choice (50 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) Solve for x in the equation $2x + 3 = -4 + 5x$.

(A) -7/3 (B) -1/3 (C) 1/3 (D) 3/7 (E) 7/3

(2) Bob and June have jobs after school. June earns 25% more than Bob does. What percent less than June does Bob earn?

(A) 10 (B) 12.5 (C) 20 (D) 25 (E) 50

(3) What is $3^{2011} + 3^{2009}$ divided by $3^{2008} + 3^{2010}$?

(A) 1 (B) 3 (C) 9 (D) 10 (E) 30

(4) If $P = (0.0123)$, $Q = (0.0123)^5$ and $R = \sqrt[5]{0.0123}$, then which of the following statements is true?

(A) $P > Q > R$ (B) $Q > P > R$ (C) $R > Q > P$ (D) $R > P > Q$ (E) $Q > R > P$

(5) If the diameter of a sphere is doubled, then the volume of the sphere is increased by a factor of:

(A) 1 (B) 2 (C) 4 (D) 8 (E) 16

(6) This year, Bob is 1/5th of the age of his Father. Next year, Bob's Father will be 6 times as old as Bob was last year. How old is Bob?

(A) 6 (B) 7 (C) 8 (D) 35 (E) 36

(7) What is x if $2^{4x-4} = 16$?

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 2 (D) 3 (E) 5

(8) Gavin wrote eight tests, each out of 100%. His average for the eight tests is 70%. If he is allowed to drop the test with the lowest value, then what is the largest possible average Gavin can have after dropping the lowest test score?

(A) 65 (B) 70 (C) 75 (D) 80 (E) Impossible to determine

(9) Ethel has pencils and pens in two pencil cases, both of which are non-empty. In one pencil case, the ratio of pencils to pens is 2:3. In the other pencil case, the ratio is 3:5. If Ethel has 20 pencils in total, what is the least number of pens that she can have?

(A) 30 (B) 31 (C) 32 (D) 33 (E) 34

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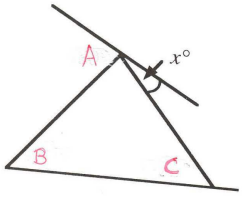
(10) In the diagram below, $A = 70°$, $B = 60°$, and $C = 56°$. What is the value of x?

(A) 46° (B) 56° (C) 60° (D) 64° (E) 70°



Questions 11-15 below are worth:

4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(11) How many three-digit numbers are there such that the number is divisible by 15, and the sum of the digits is 15?

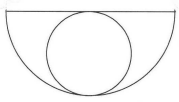
(A) 4 (B) 8 (C) 13 (D) 15 (E) 42

(12) How many numbers from 1 to 1000 have no digits that are prime?

(A) 125 (B) 216 (C) 334 (D) 500 (E) 875

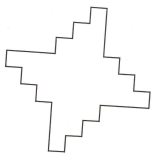
(13) The small circle shown below is the largest circle that can be drawn inside the semicircle. If the area of the small circle is 10 cm^2, what is the area of the region inside the semicircle, but outside the small circle, in cm^2?

(A) 5 (B) 5π (C) 10 (D) 10π (E) 20



(14) In the diagram below adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The perimeter of the shape is 108. What is the area?

(A) 108 (B) 216 (C) 297 (D) 300 (E) $(108)^2$



(15) If $N = (2^{p+7})(5^q)(7^3)$ is a perfect cube where p and q are positive integers, then the smallest possible sum for $p + q$ is:

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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School: _____

Full Solutions (50 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. If I take a three-digit number abc and multiply it by its reverse, cba, I get 100147. What are the two numbers?

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2. Every day June's father drives from home to pick her up at school. The way from home to the school is a long, straight road and there is no other traffic, so June's father can drive at the same speed all the time. He always leaves at exactly the right moment so that he will arrive at the school just as classes are ending. One day there was an alligator in the school and all of the students were sent home one hour early. June started walking home along the usual road. Her father met her along the way, turned around and drove her home. (Don't worry about the time involved in stopping and turning around.) They arrived home 15 minutes earlier than usual. How long did June spend walking before she got picked up?

Name: _____

School: _____

3. Let

$$S = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9} \times (100^{2008} - 1).$$

What is the sum of the digits in S?

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4. Show that the number of positive integer divisors of a perfect square is odd. (A perfect square is any number of the form a^2, e.g. $1, 4, 9, 16, 25, \ldots$)

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5. You have a chain consisting of 23 gold links. You arrive in Whammyland and discover that gold links are used for currency there. Unfortunately, merchants never provide change, so it would be nice if you could come up with exact change for every possible amount from 1 to 23 links. Explain how you can do this by cutting just two links out of the chain. (As an example, if you remove the fifth link, you have chains of size 1, 4, and 18, and you can pay exactly for something worth 5 links using the 1 and 4 chains, but you cannot pay exactly for something worth 6 links.)