Name: ____________________

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Multiple Choice (70 Marks)

Place all answers in the multiple choice boxes on the front page of the answer booklet.

Questions 1-10 below are worth:

3 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

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(1) If $(1/3)x = 15$, then $(5/9)x =$

(A) 10 (B) 15 (C) 25 (D) 5 (E) 9

(2) If $x = 1/9$, which of the following has the largest value?

(A) x (B) x^2 (C) $\frac{1}{\sqrt{x}}$ (D) $1/x$ (E) $\sqrt{x}$

(3) The least value of x that makes $\frac{27}{x-3}$ an integer is

(A) -27 (B) -24 (C) -3 (D) 0 (E) 6

(4) One angle of a triangle is three times the second angle, and the third angle is $60°$. The smallest angle, in degrees, is

(A) 45 (B) 60 (C) 20 (D) 30 (E) 15

(5) The value of $\sqrt{12 \times \sqrt{9}}$ is

(A) 3 (B) 4 (C) 6 (D) 9 (E) 12

(6) If $\frac{a}{b} = \frac{c}{d}$, where a, b, c, d are positive real numbers, and $c \neq d$, then which of the following is NOT true?

(A) $\frac{b}{a} = \frac{d}{c}$ (B) $\frac{a+b}{b} = \frac{c+d}{d}$ (C) $a^2d^2 = b^2c^2$ (D) $\frac{a}{d} = \frac{b}{c}$ (E) $\frac{a}{c} = \frac{b}{d}$

(7) Michael had an average score of 45 on his first eight economics courses, and an average score of 41 on his first nine economics courses. What score did he receive on his ninth course?

(A) 41 (B) 9 (C) 37 (D) 33 (E) 45

(8) A rectangle is increased in size. Its width is increased by 10% and its length by 30%. How much has its area increased?

(A) 30% (B) 31% (C) 40% (D) 43% (E) 63%

(9) The sum of seven consecutive positive integers is 105. The largest of the integers is

(A) 13 (B) 15 (C) 16 (D) 17 (E) 18

(10) The number of solutions (x, y) to the equation $x + 3y = 100$, where x and y are positive integers, is

(A) 33 (B) 35 (C) 50 (D) 100 (E) 34

Questions 11-20 below are worth:
4 marks for a correct answer
1 mark for a blank answer
0 marks for an incorrect answer.

(11) Determine the number of different pairs (x, y) that satisfy both $x + 2y = 4$ and $x + y^2 = 3$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(12) The fourth power of $\sqrt{1 + \sqrt{1 + \sqrt{1}}}$ is

(A) 5 (B) 7 (C) $\sqrt{2} - \sqrt{3}$ (D) $2 - \sqrt{3}$ (E) $3 + 2\sqrt{2}$

(13) A square is said to be inscribed in a circle if its four corners lie on the circle. A square is said to circumscribe a given circle if its sides are each tangent to the circle (the circle fits tightly inside the square, touching all sides). Given a circle, what is the ratio of the side length of an inscribed square to that of a circumscribing one?

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{3}$ (C) $\sqrt{6}$ (D) $\frac{1}{\sqrt{3}}$ (E) $2\sqrt{2}$

(14) One morning, three aardvarks set out in different directions to raid ant hills. At the end of the day, they meet again, and, comparing notes, find that they have pillaged 14 ant hills between them. Which of the following statements must be true?

(A) An aardvark raided exactly three ant hills. (B) An aardvark raided fewer than three ant hills.
(C) An aardvark raided more than four ant hills. (D) An aardvark raided more than five ant hills.
(E) Two of the aardvarks each raided at least three ant hills.

(15) Two gear wheels are in contact. One of them, Wheel A, has 21 teeth. The other one, Wheel B, has 35 teeth. How many complete revolutions must Wheel B make before the two wheels are in their original positions again?

(A) 3 (B) 5 (C) 7 (D) 21 (E) 735

(16) If $f(x) = 2^x$, then 16^8 is

(A) $f(f(3))$ (B) $f(12)$ (C) $f(4^8)$ (D) $f(f(5))$ (E) $f(f(f(3)))$

(17) A triangle has integer sides and perimeter 23. What is the largest possible length of any side?

(A) 6 (B) 7 (C) 11 (D) 13 (E) 21

(18) A snail at the bottom of a well 20 metres deep starts slithering up towards the top. During the day, the snail gets two metres closer to the top, but then slides one metre back down during the night. How many days does it take to get to the top of the well?

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

(19) How many positive integers no larger than 1000 are multiples of 3, but not multiples of 4?

(A) 250 (B) 166 (C) 333 (D) 666 (E) 83

(20) Four points are located on a line. The distances between pairs of points are all positive integers, and from smallest to largest are $a, b, c, d, e, 10$. What is the largest possible value of $a+b+c+d+e$?

(A) 20 (B) 27 (C) 28 (D) 30 (E) 31

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Full Solutions (30 Marks)

Place your solutions to these questions in the space provided. Each question is worth 10 marks.

You must show sufficient work to receive full marks, but if you do not completely answer a question you may still receive partial marks for showing work. So **show your work!**

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1. We have four concentric circles as shown. If each of the labelled regions has the same area, and the radius of the outermost circle is 50 m, what is the radius of the innermost circle?

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2. (a) A positive integer n will be said to be *super* if whenever $n = a + b$, with a and b positive integers, then either a or b has at least one digit in common with n. (For example, 82 is not super, since $82 = 43 + 39$, and neither 43 nor 39 has an 8 or a 2 in it.) Show that 2010 is super.

(b) A positive integer n will be said to be *superduper* if, whenever we write n as a sum of two or more positive integers, at least one of the integers in the sum has a digit in common with n. (For example, 2010 is not superduper since $2010 = 5 + 5 + 5 + \cdots + 5$, a sum of 402 fives, and none of these numbers in the sum contains a 0,1 or 2.) Find a superduper number that is larger than 10000 but smaller than 100000. Be sure to explain why it is superduper. (No credit will be awarded for a guess.)

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3. How many 5-digit numbers consist of 5 different digits, and include the digit 3? (Note that the first digit may not be zero, but the others could be.)