Junior Individual Solutions - NWO High School Mathematics Competition 2009

Multiple Choice Questions.

1. C.
2. D.
3. B.
4. D.
5. D.
6. E.
7. D.
8. C.
9. D.
10. B.
11. C.
12. D.
13. B.
14. C.
15. A.

Full Solution.

1. If Karol is telling the truth, then everyone else is lying. But then Donald is telling the truth, giving us a contradiction. Therefore, Karol is lying about being innocent. This means that Karol is the guilty one.

2. The cost of a jelly bean, in pennies, must divide 455, 154 and 110. But the only positive integer dividing all three of these is 1. Therefore, each jelly bean costs 1 cent.

3. Let r be the original radius. Then the original circumference is $2\pi r$, so the enlarged circumference is $2\pi r + 1$. The enlarged radius is, therefore, $(2\pi r + 1)/(2\pi) = r + (1/(2\pi))$, so the radius has increased by $1/(2\pi)$ cm.

4. Consider the movement relative to the water. As John moves away from the hat, he does so at speed v relative to the water, and as he moves toward the hat, he does so at speed v relative to the water. As the hat is fixed relative to the water, his time moving away equals his time moving toward the hat. Thus, he takes a total of 40 minutes after losing the hat. In that time, the hat has moved 1 km, so the speed of the water relative to the shore is $1/(40/60) = 1.5$ km/h.

5. After each stage, the number of red balls stays odd, because we either leave the total the same, or decrease it by 2. Thus, there can never be zero red balls. The last ball must be red.