2006 Thunder Bay High School Mathematics Competition

Junior Individual Answers

For complete solutions to any of the problems or for discussions of any of the problems, email ryanholm72@hotmail.com.

Note to markers: For the full solution questions students are expected to provide much more detail than I have here to receive full marks; this is just a reference sheet.

Note to students: The solutions provided here are only *sketch solutions*. Steps have been omitted and the solutions are meant to guide any student who has made a valiant attempt at solving the problem.

Multiple Choice:

1. A	9. B
2. B	10. B
3. B	11. B
4. C	12. E
5. C	13. B
6. B	14. A
7. D	15. C
8. A	

Full Solution (Answers and sketch solutions):

1. 14. The consecutive integers are 11, 12, 13, 14, 15. Hence, the average of the three largest is (13 + 14 + 15)/3 = 14.

2. 20. These can easily be systematically listed OR the number must start and end with the same digit. Hence it must start or end with 1 or 2. For each of these, any digit may occupy the middle position. Hence, there are $2\times10=20$ such positive integers.

3. 666. If his notes had 99 pages he would need $9+2\times90=189$ pages. If his notes had 999 pages he would need $9+2\times90+3\times900=2889$ pages. Hence, the number of pages has three digits. If x is the number of pages, then $189+3(x-99)=1890$ which gives $x=666$.

4. $\frac{9}{4}$. Since ABC is an acute triangle, triangles ABE and CBD are similar. Hence, BC/AB=DC/AE and this gives us that BC = $\frac{15}{4}$ by substituting in known values. Now using Pythagorean Theorem on triangle CBD we get that DB = $\frac{9}{4}$.

5. 56. Look at the last two digits, which we call m, of 2006^n. We can easily find these for the first few values of n by repeatedly multiplying 2006. This isn't as cumbersome as it seems because in order to find the last two digits of the repeated product we don't actually need to carry out the entire multiplication, we just need to carry out the multiplication on the last two digits. If n = 1, m = 06. If n = 2, m = 36. If n = 3, m = 16. If n = 4, m = 96. If n = 5, m = 76. If n = 6, m = 56. If n = 7, m = 36 and hence m will start to repeat in cycles of 5 (36, 16, 96, 76, 56). Since $2006=5\times401+1$ we thus have that m = 56 for n = 2006.