Lecture Notes – Week 2 PART 2

STRUCTURE OF THE EARTH: SEISMOLOGY

STRESS WAVE VELOCITY

Reading: Fowler, Chapter 4.1 & 4.2 (skip 4.2.8)

Objectives:

- Continue discussion of stress waves, focusing on how fast they travel through materials
- Discuss selected geophysical applications of knowledge of wave speeds

Body waves:

Recall that stress waves are vectors, i.e. they have a magnitude and direction. 'Velocity' is the vector quantity that describes the magnitude (or *speed*) and direction.

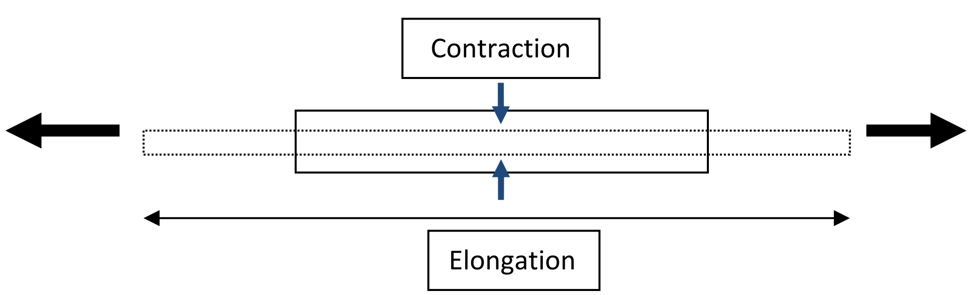
P-waves:

- P-wave speed (V_P) depends on the Young's Modulus and density of a material:

$$V_P = \sqrt{\frac{E}{\rho}}$$

- This is the one-dimensional formulation, based only on the change in length implied by Young's modulus
- Values of V_P range widely: approximately 340 m/s in air, 20-300 m/s in snow, 1500 m/s in water, 5000 m/s in granite (most rocks are between 1500 m/s and 7000 m/s)

A three-dimensional treatment of P-wave velocity requires that we consider contraction perpendicular to the elongation (i.e. a stretched bar gets thinner as it gets longer).



Poisson's Ratio = v = Contraction/Elongation ≈ 0.25

Bulk Modulus (k) is the stiffness of a material in hydrostatic compression:

$$k = \frac{E}{3(1-2\nu)}$$

And *Shear Modulus (μ) is the stiffness in shear:*

$$\mu = \frac{E}{2(1+\nu)}$$

- Using Poisson's Ratio we can re-write the P-wave speed as:

$$V_P = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}$$

- ***Note:*** *For liquids μ = 0, so*

$$V_{P-liquid} = \sqrt{\frac{k}{\rho}}$$

S-waves:

- S-wave speed (V_S) depends on the shear modulus (μ) and the density of a material:

$$V_S = \sqrt{\frac{\mu}{\rho}}$$

- There is no volume change for shear waves, so no 3D treatment is required, Poisson's Ratio not required
- ***Note:*** *For liquids μ = o so* $\boldsymbol{V_S = 0}$ ***(no shear waves in liquids)***

$V_P > V_S$:

- Two ways to show that P-waves are faster than shear waves:

$$\mu = \frac{E}{2(1+\nu)}$$

- Therefore ***E*** must be greater than **μ**
- **Therefore** $V_P = \sqrt{\frac{E}{\rho}} > V_S = \sqrt{\frac{\mu}{\rho}}$

Or use

$$V_P = \sqrt{\frac{k + \frac{4}{3}\mu}{\rho}}$$

Compared to $V_S = \sqrt{\frac{\mu}{\rho}}$

Where *k* must be greater than 0 (otherwise the material would expand when you crushed it

Therefore the $k+4/3\mu$ term must be greater than μ, which makes $V_P > V_S$

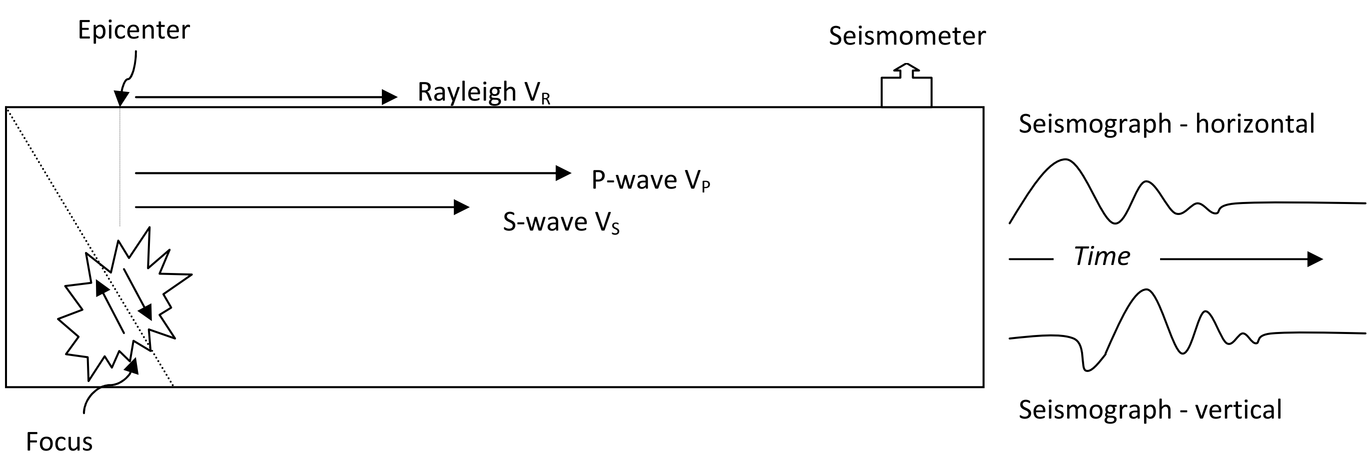
Surface Wave Speeds:

- Focus on Rayleigh Wave Speed V_R
- V_R is related to V_S by Poisson's Ratio for the material
- For most well behaved materials Poisson's Ratio is approximately 0.25, and

$$V_R \approx 0.9 V_S$$

- **Note:** Surface waves attenuate with distance must slower than others, so they can carry more energy further

APPLICATIONS OF WAVE SPEEDS: EARTHQUAKE ARRIVAL TIMES



- Use known differences in wave speeds to calculate location of epicenter relative to seismometer (subject to assumptions)

Wave Travel time (from source) = distance/speed

Wave Arrival time (at seismometer) =

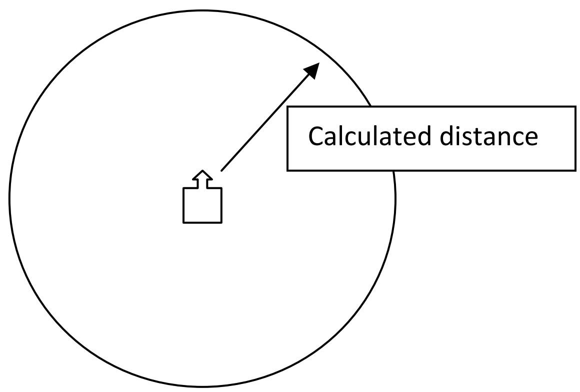
$$\mathbf{T_0 + Distance/V_P} \quad \text{(for P-wave)}$$

$$\mathbf{T_0 + Distance/V_S} \quad \text{(for S-wave)}$$

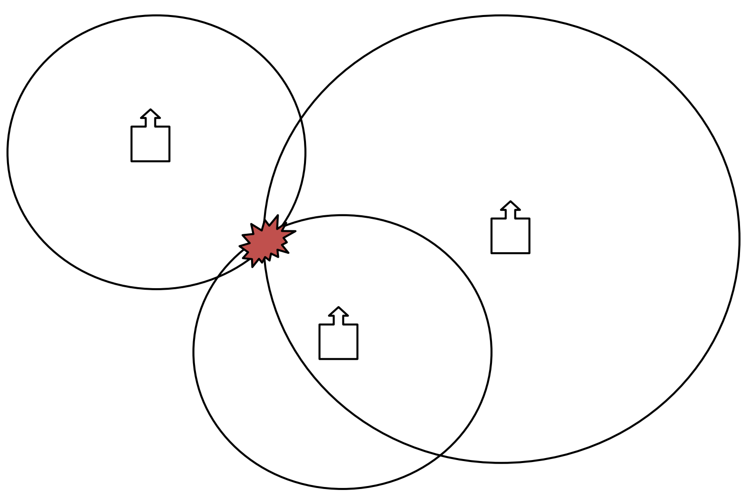
Since T_0 and Distance are the same in both,

ΔTravel Time = (Distance/V_P) – (Distance/V_S)

- Solve for Distance, and we have the distance from the epicenter to the seismometer
- But, on the earth's surface that only tells us that the epicenter is somewhere on a circle with radius of the calculated distance around the seismometer:



- If we have three or more seismometers in different locations, we can triangulate and determine the precise location of the epicenter:



- Remember that this is an oversimplified approach that treats the earth as homogenous, isotropic, and **FLAT!**
- **Also assumes that 'source' (earthquake etc.) was at the surface**
- Empirical corrections are applied to travel times and distance calculations to include these factors, often using waves reflected and refracted from subsurface earth layers – more on this later

First Motion Studies:

- Also known as Push-Pull analysis
- Simplified example considers the form of the first shear displacement to arrive at the seismometer – but of course this can also be done for P-wave arrival etc.