

## Problems

# 1. Alice, Bob, Carrie, David, Ellen, and Frank are going to sit in a row. Bob wants to sit adjacent to Alice, and Carrie wants to sit adjacent to David. In how many different orders they can be arranged under such conditions?

# 2. Among Aloysius, Bartholomew, Claudius, Diogenes and Ed, there is one, and only one, spy. Each person makes one statement. The spy will tell the truth, and so will exactly one other person. The following statements are given in the following order:

Aloysius: Diogenes is going to lie.

Bartholomew: Diogenes is not the spy.

Claudius: The spy is either Aloysius or Diogenes.

Diogenes: The spy has already made a statement.

Ed: I am the spy.

Who is the spy?

# 3. Helen has two dice. One is the standard six faces die, while the other one is a special die with two faces with "3", one face with "4", three faces with "6", and no faces with "1", "2", or "5". She throws the two dice simultaneously. What is the probability that the sum of the two faces is 8?

# 4. What is the greatest number of  $90^\circ$  interior angles that a decagon can have?

# 5. A square ABCD has side length 2. A semicircle is drawn inside the square so that the diameter (flat side) of the semicircle coincides with the side AB of the square. A line is drawn from the corner C of the square tangent to the semicircle, and intersecting the side AD at a point E. Find the area of the triangle CDE.

# 6. A right triangle has an angle of  $54^\circ$ . A regular polygon with  $n$  sides is positioned inside the triangle so that three of its sides lie along the sides of the triangle. What is the smallest possible value of  $n$ ?

# 7. Show that any positive integer  $k$  can be expressed as  $k = \frac{mn+1}{m+n}$ , where  $m, n$  are positive integers.

# 8. Let  $A = 2222^{2221}$ ,  $B$  be the sum of all digits of  $A$ ,  $C$  the sum of all digits of  $B$  and  $D$  the sum of all digits of  $C$ . Find  $D$ .

# 9. Let  $C$  be the unit circle  $x^2 + y^2 = 1$  and  $P$  be the point  $(2, 0)$ . Let  $S$  denote the set of points that arise as the midpoint of a line segment from  $P$  to some point on the circle  $C$ . Determine the set  $S$ , and sketch it.

# 10. Find the maximum value of  $ab^2(12 - a - b)$  where  $a, b \geq 0$ .

# 11. A pair of positive real numbers  $x$  and  $y$  satisfy  $1 + \log_2 y = \log_5 x = \log_{10}(x + y)$ . Find  $(1/x + 1/y)$ .

# 12. Three smart students Adam, Ben, and Carolina are guessing two integers  $x$  and  $y$ , such that  $1 < x < y < 10$ . Adam is given the sum  $x + y$ , Ben is given the product  $xy$  and Carolina is given  $(x + y) + xy$ . Each student knows what information the other students have been given. The 3 students are asked to write down their answer if they know  $x, y$  for sure. But no one knows what  $x$  and  $y$  are. After knowing that no one knows the answer in the first round, all of the 3 students know the correct answer quickly in the second round. What are these two numbers  $x$  and  $y$ ? Show your reasoning.