A system of nonlinear equations with application to large deviations

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In this talk, we first show the existence of solutions to the following system of nonlinear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_{11}\frac{1}{x_1} + b_{12}\frac{1}{x_2} + \cdots + b_{1n}\frac{1}{x_n}, \\ a_{21}\frac{1}{x_1} + a_{22}\frac{x_2}{x_1} + \cdots + a_{2n}\frac{x_n}{x_1} = b_{21}x_1 + b_{22}\frac{x_1}{x_2} + \cdots + b_{2n}\frac{x_1}{x_n}, \\ \cdots\cdots \\ a_{k,k-1}\frac{1}{x_{k-1}} + \sum_{\substack{1\le j\le n \\ j\neq k-1}} a_{kj}\frac{x_j}{x_{k-1}} = b_{k,k-1}x_{k-1} + \sum_{\substack{1\le j\le n \\ j\neq k-1}} b_{kj}\frac{x_{k-1}}{x_j}, \\ \cdots\cdots \\ a_{n,n-1}\frac{1}{x_{n-1}} + \sum_{\substack{1\le j\le n \\ j\neq n-1}} a_{nj}\frac{x_j}{x_{n-1}} = b_{n,n-1}x_{n-1} + \sum_{\substack{1\le j\le n \\ j\neq n-1}} b_{nj}\frac{x_{n-1}}{x_j}, \end{cases}$$

where $n \geq 2$ and $a_{ij}, b_{ij}, 1 \leq i, j \leq n$, are positive constants. Then, we make use of this result to obtain the large deviation principle for the occupation time distributions of continuous-time finite state Markov chains with finite lifetime. The talk is based on a recent joint paper with Zechun Hu and Jing Zhang.