THE ROLE OF CO-CONSTRUCTION OF THE CLOCK MODEL IN THE DEVELOPMENT OF FRACTIONAL UNDERSTANDING IN A GRADE 4/5 CLASSROOM

By

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Chapter 1: Introduction

Context

Mathematics education is changing. There are a variety of factors spurring on this transformation, including knowledge gained from research, the prevalence of technology, and the influence of the National Council of Teachers of Mathematics (NCTM), a U.S.-based organization composed of mathematics educators from the United States and Canada (Battista, 1999; Van de Walle, Folk, Karp & Bay-Williams, 2011).

In the United Sates the NCTM published its *Curriculum and Evaluation Standards for School Mathematics* in 1989, closely followed by the publication of a companion document, *Professional Standards for Teaching Mathematics* in 1991. Implementation of these two influential documents marked the beginning of the "reform era" in mathematics education. The NCTM-influenced new curriculum documents published by the Ontario Ministry of Education followed, first in 1997, and then replaced by *The Ontario Curriculum, Grades 1-8: Mathematics, 2005.* These documents advocated shifts in the classroom environment, reflecting the belief that teaching needed to change in order to improve student learning. What was it about the teaching of mathematics that needed to change?

Numerous studies have shown "traditional" methods of teaching mathematics to be ineffective, and even harmful, because they discourage mathematical reasoning (Battista, 1999; Kamii & Dominick, 1998). The traditional view sees mathematics as a set of computational skills and procedures to be learned. In a traditional mathematics class the teacher delivers the lesson; the students copy and practice. In contrast, Battista (1999) summarizes "reform" teaching methods as occurring in classrooms where:

> teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write, and discuss mathematics; and to formulate and test the validity of personally constructed mathematical ideas so that they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects — as well as formal mathematical and logical arguments — to convince themselves and their peers of the validity of their solutions. (pp. 427-428)

In addition to realizing that there were difficulties caused by traditional teaching methods and that classroom practice should be altered, researchers were drawing on the theory of *constructivism*.

Constructivism. Constructivism is a theory about knowledge and learning. Knowledge is viewed as being personally constructed by learners as they try to make sense of situations, not as existing independently and absolutely as "truths to be transmitted or discovered" (Fosnot, 2005, p. ix). Students learn — construct new knowledge based on prior knowledge — by reflecting on, or actively thinking about, an idea, rather than passively absorbing it unaltered from the teacher. Because constructivism is about learning, it is a theory with important implications for the classroom. Fosnot and Perry (2005) contend that "teachers need to allow learners to raise their own questions, generate their own hypotheses and models as possibilities, test them out for viability, and defend and discuss them in communities of discourse and practice" (p. 34).

Refinements to Reform Instruction. Researchers continue to refine the conceptualization of effective reform instruction initially outlined in the NCTM *Standards*. As one of a number of refinements, some shifted their focus on the importance of using mathematical manipulatives as a means to deepening student understanding to the contention that models must be developed in a familiar context through the activity of modeling (Gravemeijer, 2002). Researchers came to believe that if manipulatives are used as models, by essentially being given to the students by the teacher rather than being co-constructed by the students together with their teachers, then students often misunderstand the mathematics; that is, they do not ‘see’ it in the manipulative in the way the teacher does (Ball, 1992). This has implications for those areas of mathematics where manipulatives as models have played a central role, and is the case with the teaching of fractions. It may be that a better choice, and introduction, of mathematical models can play an important role in the improvement of instruction and understanding of fractions, a particularly poorly understood area of mathematics.

Purpose of the Study

The purpose of this study is to investigate the co-construction and use of the clock model by students in the context of a unit on fractions taught with a reform method approach. This study is designed to examine the process and impact on student understanding of a teacher and her students co-constructing a model for comparing and adding fractions. A pre-test will be administered as a diagnostic tool to ascertain the students’ understanding and use of tools and representations prior to instruction. The unit on fractions will be taught with an emphasis on problem solving and student-generated solutions. The mid-test will be administered part way through the unit, before the clock model is co-constructed. The results of the post-test will be compared with the mid-test and pre-test to determine the impact resulting from the use of the clock model.

Research Question

What is the impact of the co-construction and use of the clock model on students' understanding of fractions?

Significance of the Study

While there is a substantial body of research pertaining to the area of learning fractions within reform mathematics instruction, there is nonetheless much more to be accomplished (Lamon, 2007), including in the area of effective modelling. As Gravemeijer (2002) contended, there is "a growing interest within the mathematics community in the role of symbolizing and modeling" (p. 7); yet there is a lack of classroom-based research on these topics. Indeed, searches of ERIC, CBCA, and Professional Development Collection education databases using the key words "fractions" and "clock" yielded only a few teaching suggestions for connecting fractions to telling time on the analog clock (Friederwitzer & Berman, 1999) and relating the hours of the day to fractions (May, 2000). Finally, an article by Chick, Tierney and Storeygard (2007) contains a description of two students' understandings as they solved traditional fraction problems using a clock face. No published research on the use of the clock model in the manner suggested by Fosnot and Dolk (2002) was located. Thus the study addresses a current gap in the literature.

This research is a case study of a class of grade 4/5 students as they learn about fractions. It will provide an in-depth exploration of their learning. This study will contribute information that is relevant to mathematics teachers and educators and will shed light on, as Gravemeijer, Lehrer, van Oers, and Verschaeffel (2002) wonder, "how to support and guide this process of knowledge construction without interfering with students' initiative and intellectual autonomy" (p. i).

Contribution to the Community

Teachers are interested in improving their practice. The classroom teacher and I will collaborate to prepare a workshop for teachers in the School Board after the study has been completed. Furthermore, the use of the lesson videotapes gathered as data in this study will contribute to in-service teacher professional development and may lead to increased student success as a result.

Limitations of the Study

Some limitations of the study should be considered, namely the design of the research project and the testing instruments. As a case study, this research will provide a rich description of the learning that occurs in one classroom. Most of the time, case studies are undertaken to understand a particular case, not to generalize to other cases (Stake, 1995). In this instance it is clear that one classroom will not be representative of all grade 4 and 5 students. This study is not designed to make a comparison between two different teaching methods or models in order to make a judgment about which might be superior. The intent is to look at the impact of the co-construction of the clock model and describe how it supports student learning.

Pre-test, mid-test, post-test, and retention test questions will be somewhat different, although designed to address the same content at the same level of difficulty. The difference in questions may affect the results in a way that is not a direct outcome of the instruction. The post-test will be part of the summative evaluation of the unit, so the students may be more motivated to perform than on the other tests. Nevertheless, the post-test is not the only source of data that will be used to draw conclusions for the study, so while there could be some impact of student motivation the conclusions will be mitigated by other sources of data.

Chapter 2: Literature Review

Introduction

Learning about fractions is a challenge for students (Lamon, 2007; Van de Walle et al., 2011). Most of the time, fractions are taught in the same way mathematics has been traditionally taught. The emphasis is on learning algorithms, which are set procedures that are followed in order to complete a calculation. For example, an analysis by Cramer, Post and delMas (2002) of commercial fourth- and fifth-grade textbook curriculum used in one school district revealed the emphasis was on gaining procedural competency, and distressingly, that "symbol manipulation seemed to be an end in itself, independent of context and physical models" (p. 139). Researchers have been concerned that students can become proficient in carrying out rote procedures but, when instruction focuses on computation rather than on understanding, may later lose this apparent mastery (Aksu, 1997). They believe that when procedural knowledge is taught before or without developing conceptual knowledge, children may not see the connection between the two and the result is that they must memorize the algorithms. Conversely, when children can instead build on their intuitive knowledge of fractions gained by personal experience and eventually come to rely less on the specific context, they continue to make sense of fractions. Once children have developed conceptual knowledge of fractions, they can connect concepts to procedures and make use of formal symbols and algorithms (Sharp, Garofalo & Adams, 2002).

Helping students develop conceptual understanding, rather than concentrating on symbols and procedures, is a general recommendation for improving the teaching of fractions. What are the specific areas that cause problems for students as well as the recommendations for remediation found in the literature?

Instructional Difficulties

Researchers have documented many problems that students have with learning fractions and have tested teaching strategies that could help students to avoid, or at least minimize, the difficulties. Six important concepts, the problems faced, and some suggested instructional solutions are examined below.

Fractional Parts are Equal-size Portions. A key mathematical idea underpinning the development of fraction concepts is the recognition that “fractional parts are equal shares or equal-sized portions of a whole or unit” (Van de Walle, 2007, p. 293). Researchers have found that children do not always realize that the whole must be partitioned into pieces of equal size (Pothier & Sawada, 1983). In their study, Reys, Kim and Bay (1999) posed a series of open-ended questions to a class of fifth graders who had recently completed a six-week unit on fractions. When asked to describe 2/5, in interviews seven out of the twenty students gave responses that revealed they held the misconception that the whole did not need to be partitioned into pieces of equal size. Three children made inaccurate circle diagrams to represent the fraction 2/5 and were not concerned that the pieces were not of equal size, even when questioned about the sections as if they were pieces of pizza. About one third of the class held a misconception about this fundamental fractions concept; clearly something was lacking in the instruction they had received.

While many would suggest beginning with students’ intuitive knowledge Pothier and Sawada (1983) found, in the context of cutting and sharing pieces of birthday cake, that some children were more concerned with making sure the number of pieces for each person was the same, rather than fairness of the size of the pieces. Therefore, surprisingly, the researchers observed that informal knowledge is not always a reliable starting point. Most other researchers and math educators, however, have found otherwise. They advocate the use of a fair-sharing context (Empson, 2002; Flores & Klein, 2005; Fosnot & Dolk, 2002; Sharp et al., 2002). With fair-sharing tasks, it is best to make use of items which can easily be subdivided, such as brownies, pizzas, sandwiches, or candy bars (Van de Walle, 2007), in a context which is personal to the children (Fosnot & Dolk, 2002; Sharp et al., 2002).

Additionally, in order to help students construct the importance of equal sized portions in fractions, Reys, Kim, and Bay (1999) suggested that teachers need to provide more opportunities for students to work with visual representations of fractions, such as shading fractional parts of shapes, locating fractions on a number line, or separating tiles, and recommend that teachers revisit these representations at the fifth-grade level in order for students to fully conceptualize fractions before they move on to procedures with fractions. Researchers with the Rational Number Project (Cramer et al., 2002) showed that the use of, and translations between, multiple physical manipulatives, symbols, pictures and contexts in initial fraction learning is effective in helping children to construct accurate mental images of fractions.

Understanding Fraction Symbols. Researchers have found that the symbolism used to represent fractions is a complex convention and can be misleading (Van de Walle, 2007). For example, in Mack's (1995) research with third and fourth graders, students "thought the numerator represented the number of wholes, or units, being considered and the denominator represented the number of parts in each whole" (p. 430). Mack tried to help students relate fraction symbols to fractions used in the context of real-world problems they had solved verbally, but it was very difficult. Students drew on previous knowledge of whole numbers rather than on their informal knowledge of fractions when writing and explaining fraction symbols. After Mack showed the children how to write mixed numbers and explained their meaning in the context of real-world situations, the children were able to write mixed numerals and stopped saying that the numerator was the number of wholes under consideration.

Since fraction symbolism is a convention, Van de Walle (2007) advised first to delay its introduction until students have worked with fractional ideas and then to simply tell students the way fractions are written, but to make the convention very clear by leading the following demonstration. He recommended displaying several collections of fractional parts to the class, including some improper fractions, having the students count the parts together, writing the correct fractional symbol for each one, and then posing two questions to the class: “What does the bottom number in a fraction tell us? What does the top number in a fraction tell us?” (p. 299). The important point is that children grasp that the top number is the counting number and the bottom number tells what is being counted. Use of the terms numerator and denominator will not help children understand the meaning of the symbolism.

Improper Fractions. Another misconception held by many students is that fractions are always less than one whole. In Mack’s (1990) study of sixth graders, she found that students were not able to identify the unit when presented with multiple units concretely or pictorially because of their belief that fractions are always less than one whole. However, when the situation was given a real-world context, such as pizzas, students were able to interpret the unit correctly. Tzur (1999) was puzzled by the inability of fourth grade students to think about fractions greater than 1. Working together with sticks of various lengths in a computer microworld, the students would change the unit after producing nonunit fractions by iterating a unit fraction. (A unit fraction is a single fractional part and has a numerator of 1; a nonunit fraction has a numerator greater than 1). Tzur related this particular observation from his teaching experiment:

> For example, they regarded the nonunit fraction produced by the iteration of 1/5 four times as 4/5 or the unit produced by iteration of 1/5 five times as 5/5, but when they iterated the same 1/5 six times, they thought about the resultant stick as 6/6 and each part was then regarded as 1/6. (p. 397)

It is likely that this misconception has its roots in the way fractions are commonly introduced in textbooks, with geometric figures displaying shaded parts of the whole, as well as in the language of fractions. Flores and Klein (2005) reminded us that the school language of fractions differs from everyday usage. In daily use, the term ‘fraction’ means a part less than a whole. In mathematics, the term ‘fraction’ refers to the quotient of two quantities. In addition, the term ‘improper fraction’ may confuse students by suggesting that there is something unacceptable about this type of fraction. It is important to give children the opportunity to count pieces, so that ¾ can be viewed “not only as 3 out of four pieces but also as 3 pieces of size ¼ each” (p. 456). This can be extended to counting 7 pieces of size ¼ each, for example. Also, the posing of fair-sharing problems such as sharing 7 brownies among four people will elicit answers that include 7/4, among others. Solving problems like this will assist students in constructing ‘improper fractions’ more easily. Van de Walle et al. (2011) recommended saying ‘fractions greater than 1’ or just simply ‘fractions’ rather than using the term ‘improper fractions’ (p. 305). And, as mentioned above, improper fractions should be included when introducing fraction symbols (Van de Walle et al., 2011). Indeed Kamii and Clark (1995) reasoned that proper and improper fractions, as well as mixed numbers, should all “be involved from the beginning so that children will think about parts and wholes at the same time” (p. 375).

Size of Unit Fractions. Children often have difficulty conceptualizing the relative size of unit fractions (Van de Walle, 2007). Mack (1990) found, for example, when sixth grade students were asked which fraction is bigger, 1/6 or 1/8, most of them chose 1/8. Their explanations for choosing 1/8 as the larger fraction suggested that they were trying to apply rules for whole numbers to fractions. When the same comparison was placed in the context of slices of pizza however, the students drew on their informal knowledge and answered with 1/6. "The inverse relationship between number of parts and size of parts cannot be told but must be a creation of each student's own thought process", as stated by Van de Walle (2007, p. 304). Alluding to constructivism, Van de Walle made the case that understanding cannot simply be 'given'; rather, it must be constructed by the learner. In order to overcome the misconception, Van de Walle suggested teachers have children go through an activity of putting a list of unit fractions in order from least to most, defending the way they ordered the fractions, and explaining their ideas by using models (p. 304).

Alternatively, Fosnot and Dolk (2002) re-asserted that the misconception can usually be avoided "when fractions are introduced right from the start within fair-sharing contexts, as division" (p. 56). They explained that when fractions are introduced as shaded parts of whole entities, as they traditionally are, this is introducing fractions as a number within a measurement model, which is quotative division. In this context "learners often assume that the greater the denominator the greater the amount" (Fosnot & Dolk, 2002, p. 56). In a fair-sharing context however, children understand (by physically sharing wholes) that the greater the number of pieces, the smaller the amount, and avoid the problem of assuming that a greater denominator signifies a greater amount.

Estimation and Comparison: The Need for Fraction Number Sense. Another difficulty that arises from traditional fraction instruction is that many students are not able to estimate a sum or correctly compare the sizes of fractions (Cramer & Henry, 2002; Cramer et al., 2002; Reys et al., 1999). This situation signifies a lack of fraction number sense: students need to develop an awareness of the approximate size of a fraction in order to estimate. Traditional textbook-based instruction does not include estimation; students are taught to rely on rote procedures such as rewriting fractions with common denominators or using cross-multiplication to be able to compare fractions.

One way to overcome the inability to estimate is to teach about important reference points, or benchmarks (Huinker, 1998). Essential fraction benchmarks are 0, ½, and 1 (Van de Walle, 2007). Fosnot and Dolk (2002) also recognized the importance of using benchmark or landmark fractions as a strategy. Teachers need to intentionally model the use of fraction benchmarks for their students and show that it is a useful process that they will use outside of math class (Reys et al., 1999).

A major study carried out by researchers with the Rational Number Project (RNP) demonstrated that there is a method of teaching fractions using manipulatives which helps children build fraction number sense. In 2002, Cramer et al. reported on their research which measured and contrasted the effects of two different curricula on fractions achievement and thinking of over 1600 fourth and fifth grade students. One group used district-approved commercial curriculum (CC); the other used the RNP curriculum. The RNP curriculum emphasized “the extended use of multiple physical models and translations within and between other modes of representation — pictures, written symbols, verbal symbols, and real-world contexts” (pp. 137-138). In the commercial curriculum the emphasis was on gaining procedural competency. Written tests and task-based interviews were used to measure achievement and reveal students’ thinking.

On the written tests, RNP students had significantly higher scores on four of the subscales: concepts, order, transfer, and estimation (Cramer et al., 2002). Unexpectedly, no significant differences were found between the groups of students on equivalence items or symbolic operations tasks. The researchers had expected CC students to outperform RNP students on operations tasks requiring exact answers, since CC students had spent significantly more time on this topic. Thus, as the authors state, "RNP students' development of procedural knowledge was apparently not impeded despite their having devoted very limited classroom time to it" (p. 138).

The interview data revealed differences between the two groups in students' thinking (Cramer et al., 2002). The RNP students displayed higher percentages of conceptually oriented answers than the CC students. They used mental models of fractions, chiefly the circle model, to determine relative sizes of fractions for the purpose of putting them in order, and also extended the use of the mental representations to estimating sums and differences. In general, the CC students relied on procedures such as finding least common denominators or cross products to obtain solutions. These results clearly reflected the curriculum used by each group. Consequently, it can be concluded that working with physical models assists children in developing fraction number sense; students created mental images of fractions which in turn helped them to understand fraction size.

Equivalent Fractions. Another crucial yet problematic area for students is equivalent fractions (Kamii & Clark, 1995). This is the first time in their mathematics learning students encounter a situation where "a fixed quantity can have multiple names (actually an infinite number)" (Van de Walle et al., 2011, p. 310). For Huinker (1998), understanding that "a specific amount can have many names" (p. 172) is a critical aspect of fraction knowledge. Fosnot and

Dolk (2002) theorized that two "*big ideas*"[1] must be constructed by children in order to understand equivalent fractions: "for equivalence the ratio must be kept constant" and "pieces don't have to be congruent to be equivalent" (pp. 136-137). In the first of these two big ideas, understanding ratio means understanding the relationship within a fraction, between the numerator and the denominator, and then extending this relationship across two fractions. In the second big idea, congruent pieces would be pieces that are the same shape and size. Two fractions can be equivalent, even if the pieces are not the same shape, as long as they are the same size. For example, in sharing submarine sandwiches, three pieces of ¼ of a sub are the same amount as ½ plus ¼ of a sub: "the quantity stays the same even though the pieces look different" (p. 56). Fosnot and Dolk's contention is substantiated in the research. An exploratory study found that students were able to explain equivalent fractions when presented to them as visual geometric area models, especially circles, but had difficulties with numerical symbolic notation (Jigyel & Afamasaga-Fuata'i, 2007). One conclusion reached by the researchers is that children need to develop their understanding of the idea of a fraction as the relationship between the numerator and the denominator.

The general approach to equivalent fractions encouraged by Van de Walle et al. (2011) is to have students use a variety of models (area, length, and set) to generate different names for fractions. Lamon (1996) suggested that partitioning activities should continue to be given throughout middle school so that students develop increasingly sophisticated partitioning strategies, rather than thinking of partitioning as a simple introductory level activity for students in the third grade only. When comparing the value of partitioning by physically cutting pieces with the value of making drawings, Lamon stated: "there is a greater chance that notions of

[1] "Big ideas" are "the central, organizing ideas of mathematics – principles that define mathematical order" (Schifter & Fosnot, 1993, p. 35).

equivalence will be visually induced in the paper-and-pencil tasks (p. 189). Even so, Kamii and Clark (1995) showed that children's figurative knowledge (based on what is observable) can be in conflict with the operative knowledge (based on relationships, which are not observable). This conflict will cause children to say that a triangular half of a square is larger than a rectangular half of a square of the same size. Students have trouble constructing the idea that one half is equal to one half if the regions are shaped differently. Lamon (2002) asserted that students are able to generate equivalent fractions using the reasoning of "unitizing". Unitizing is the "process of mentally constructing different-sized chunks in terms of which to think about a given commodity" (p. 80). Lamon wrote that one advantage of unitizing is that children can reason in this way about fractions "even before they have the physical coordination to be able to draw fractional parts accurately" (p. 82). There does not seem to be a consensus in the literature on the best way to teach equivalent fractions.

Summary of Instructional Difficulties and Remedies. Throughout the discussion of difficulties students have learning fractions a number of recurring themes on remediation have emerged. Firstly, children bring informal knowledge to class and learning should build on this strength as much as possible. For example, fair-sharing contexts offer a rich environment for introducing fractions and partitioning activities should be given to children from the beginning and continue through to the middle grades. There are recommendations to include mixed numbers and fractions greater than 1 from the start, while avoiding the use of the term "improper fraction". Posing problems that draw out a variety of student-generated strategies and representations should be the core of fraction instruction. Working with benchmark fractions can help students learn fraction number sense. Opportunities to utilize a variety of different visual representations of fractions should be provided to children. Manipulatives and models have also been discussed as potential solutions to the problems, with mixed reviews. I single these out for detailed discussion as our understanding of their use and worth has been refined since the early days of reform.

Manipulatives

It is commonly agreed, even taken for granted, that the use of manipulatives is an essential component in good elementary mathematics instruction (Ball, 1992; Thompson, 2002). However, researchers have found that there is no guarantee that the use of manipulatives will have the desired effect on students' learning (Baroody, 2002; Clements & McMillen, 2002; Thompson, 2002). A widely held assumption is that mathematics is embodied and evident in the physical materials themselves, but this is simply not the case: Mathematical ideas are constructed in each human mind (Clements & McMillen, 2002). Each situation is open to interpretation and the student may not see what the teacher sees when looking at the same item (Ball, 1992; Thompson, 2002; von Glasersfeld, 2005). Another fallacy is the "notion that understanding comes through the fingertips" (Ball, 1992, p. 17), as if by physically manipulating objects students will learn mathematical concepts. Many researchers believe, however, that the learning that happens through the use of manipulatives is most successful when students reflect on their actions with the manipulatives, thereby linking ideas and making connections (Clements & McMillen, 2002; Stein & Bovalino, 2001; Thompson, 2002). The students' learning experience must be guided, but not too restrictive, lest the activity become a matter of mindlessly following a procedure (Baroody, 2002; Stein & Bovalino, 2001).

Teachers need support in using manipulatives in this way in their classrooms (Ball, 1992). Those teachers who are trained in the use of manipulatives and have prepared well enjoy the most successful lessons (Stein & Bovalino, 2001). When planning a lesson, it is best if teachers consider what they want their students to understand, not what they want the students to do (Thompson, 2002). One of the most important aspects to consider is "that the experience be meaningful to students and that they become actively engaged in thinking about it" (Clements & McMillen, 2002, p. 260). A final point to take into consideration is that "certain computer manipulatives may be more beneficial than any physical manipulative" (Clements & McMillen, 2002, p. 260) for a variety of reasons.

Perhaps the reason that manipulatives do not work the magic that educators had hoped for can be found by exploring three different kinds of knowledge. Kamii and Warrington (1999) discussed the teaching and learning of fractions in light of Piaget's three types of knowledge: *social* (or conventional) *knowledge*, *physical knowledge*, and *logicomathematical knowledge*. The traditional method of teaching can be categorized as transmitting social knowledge, since the intent of the process is to pass the knowledge of algorithms from teacher to student. Manipulatives were recommended as a remedy for the shortcomings of traditional teaching methods with the hope that they would increase conceptual learning (Kamii & Warrington, 1999). As has been discussed, having students work with manipulatives, which is building physical knowledge, does not necessarily achieve the desired outcome. The thing to do is to focus on the development of reasoning, or logicomathematical knowledge, which "develops out of children's own mental actions" (Kamii & Warrington, 1999, pp. 85-86).

Models for Fractions

Van de Walle (2007) gave the following definition of a model: a "*model for a mathematical concept* refers to any object, picture, or drawing that represents the concept or onto which the relationship for that concept can be imposed" (p. 31). Visual and physical models of fractional parts and wholes "can help students clarify ideas that are often confused in a purely symbolic mode" (p. 295). Van de Walle described three main categories of models for fractions: region or area models, measurement or length models, and quantity or set models. Area models encompass circular pies, rectangular regions, geoboards, grids or dot paper, pattern blocks, and paper folding. Length models include fraction strips, Cuisenaire rods, line segment drawings, number lines, and folded paper strips. Set models are groups of items such as counters or drawings of sets. It is important to use a variety of models — the same activity with different models will be different from the students' perspective (Van de Walle, 2007), although experts (e.g. teachers) "seem to view all modes of presentation of information as equivalent" (Bright, Behr, Post & Wachsmuth, 1988, p. 230).

Circle models are used most commonly (Van de Walle, 2007), but the use of circle models alone may unintentionally reinforce whole number thinking (Moss & Case, 1999). On the other hand, there has been great success with use of the circle model in fostering fraction number sense, as outlined above (Cramer & Henry, 2002; Cramer et al., 2002). Kamii and Clark (1995) hold a contrasting view, however, stating that "we would not provide any fraction circles and encourage children, instead, to make their own drawings" (p. 376). In this way, the drawings represent the children's own knowledge and understanding, unlike drawings "presented in textbooks, which represent someone else's thinking" (p. 376).

The use of the number line, another model in teaching fractions, is encouraged by many researchers and is connected to the real-world context of measuring (Van de Walle et al., 2011). After completing clinical and large-group teaching experiments with fourth and fifth graders on the subject of representing fractions and ordering fractions on number lines, Bright et al. wrote, "number line instruction is difficult" (1988, p. 227). Students struggled with representing equivalent fractions on a number line, did not successfully transfer knowledge to slightly different situations, and appeared to have had trouble connecting symbolic and pictorial information. It may be that the way the number line was introduced to these students was ineffective — it was presented to the students rather than developed in a meaningful context.

The experimental curriculum created by Moss and Case (1999) was aimed at helping children in grade four develop concepts of rational numbers overall, not fractions alone. The curriculum was built on a linear measurement model they refer to as the number ribbon, which emphasized continuous quantity, measurement, and proportion rather than discontinuous quantity and counting. Moss and Case encouraged the children to work with landmark percent, decimal, and fraction equivalences. The program began with a beaker of water as a visual prop; the height of the beaker was the number ribbon model. In a question where the beaker was to hold a specific amount of water, the number ribbon functioned as a double number line, double because there are two numbers to compare: percentages and millilitres. Fosnot and Dolk (2002) developed the use of the double number line for the purpose of addition and subtraction of fractions with unlike denominators, as well as with percentages. It is interesting to note that Moss and Case (1999) witnessed their students spontaneously using a money model in the context of calculating percentages; Fosnot and Dolk (2002) also identified the money model and its usefulness for developing landmark decimals.

Another model proposed by Fosnot and Dolk (2002) is the clock model. They suggested it would be helpful for "adding and subtracting fractions like fourths, thirds, sixths, and twelfths" (p. 89). The clock model functions as a double number line by allowing common fractions to be converted into whole numbers of minutes, operated on, and then converted back into fractions. Chick et al. (2007) reported on using a clock face "as a way for students to think about fractional parts of a whole – as twelfths and their equivalent fractions" (p. 52). The students in Chick's grade five class were given worksheets and were shown how to draw line segments from the centre of the clock face to the numerals 1 to 12 and to draw arcs to represent the motion of the minute hand. Samples of the work of two children in the class showed that they shaded in the sections when adding fractions, which is using the clock face as an area model. The children did not convert the fractions of an hour into minutes to solve problems. In the article, Chick et al. (2007) reported that close observation of two students as they worked with clock fractions revealed differences in the students' understanding. One student did not appear to have a conceptual understanding of fractions. She relied on memorized procedures for dealing with fractions, had difficulty identifying parts of the whole on diagrams of the clock face, and did not appear to understand fractions as relationships. The other student had a conceptual understanding of fractions which was revealed in the diagrams and symbols she produced, but interestingly had not been demonstrated on previous tests involving computations. While both Fosnot and Dolk and Chick et al. made use of a clock model, they do so in very different ways that render these, in essence, different models: one a linear model based on the double number line and the other a circular area model.

The mixed reviews of the effectiveness of each of these models may be related to the context in which the model was used (particular models may work better to model particular problems), and also, how the model was introduced — was it given by the teacher or developed with the students?

The Activity of Modeling

Researchers have recently begun to carefully explore the latter issue of model introduction in mathematical lessons. In this view of mathematical models, "the use of pre-designed models is replaced by the activity of modeling" and "modeling is primarily seen as a form of organizing, within which both the symbolic means and the model itself emerge" (Gravemeijer, 2002, p. 7). Similarly, Fosnot and Dolk (2002) described mathematical models as "mental maps of relationships that can be used as tools when solving problems" (p. 90). In their view, the effective introduction and use of models has three stages. Models begin as the child's representations of *action* in a situation, develop into representations of the situation, and finally mature into a symbolic representation of *mathematizing*[2] (Fosnot & Dolk, 2002). Fosnot and Dolk insist that models must be developed within a rich context and learners must construct models for themselves; models cannot be transmitted (Fosnot & Dolk, 2002).

A practical question that arises out of this view of models and modeling is how does one teach students to construct models for themselves? Van Dijk and colleagues (van Dijk, van Oers & Terwel, 2003; van Dijk, van Oers, Terwel & van den Eeden, 2003) studied and compared two differing approaches, (a) providing ready-made models to students, and (b) guiding students in the co-construction of models. Qualitative observations showed that a child who was provided with models had difficulty using them, whereas a child who co-designed her own models had no difficulty making use of them (van Dijk, van Oers & Terwel, 2003). In a larger study, children who were taught to design models outperformed children who were directly provided models (van Dijk, van Oers, Terwel & van den Eeden, 2003). Researchers have investigated different problems, contexts, and calculations which promote children's creation of models as a means of solution. Fosnot and Dolk (2002) recounted a *minilesson* that could serve as the foundation for the co-construction of the clock model. In the minilesson, the teacher posed a carefully designed series of fraction addition problems, embedded in a context that involved the use of time, with

[2] Mathematizing means "interpreting, organizing, inquiring about, and constructing meaning through a mathematical lens" (Fosnot & Dolk, 2002, p. 18).

the purpose of eliciting responses from the children that would assist them in developing the use of the clock model.

Summary and Implications

There is no question that models are crucial for solving problems. Some researchers contend that effective models are those that children construct for themselves (Fosnot & Dolk, 2002). If children can co-construct a model as a class together with the teacher, it is more beneficial than attempting to transmit the model directly from the teacher to the student (van Dijk, van Oers, & Terwel, 2003; van Dijk, van Oers, Terwel & van den Eeden, 2003). If these more recent theories about the effective development of mathematical models are applied with a specific, less-studied, model in the instruction of fractions — the clock model — what kind of impact would it have on student understanding of and facility with this often challenging area of mathematics?

Chapter 3: Methodology

Research Design

This research project is a qualitative case study on the effect of the co-construction of the clock model on students' understanding of fractions in the context of reform mathematics instruction. As such, it is a *single instrumental case study* (Creswell, 2007), because in-depth understanding of the issue will be illustrated with one bounded case, the teacher and students in a class of Grade 4/5 students. This research methodology is similar to Garrett (2010). A pre-test, mid-test, intervention, post-test, retention test sequence will be employed.

Research Sample

The project will be carried out with a *convenience sample* (Creswell, 2007). The research will be conducted with a Grade 4/5 class in a School Board in Northwestern Ontario.

Procedure

Ethics approval is required from Lakehead University, the school board, and the principal of the school where the study will be conducted. Since student data are being used, introductory letters and permission forms will be sent to parents and guardians (Appendices A and B); students will receive their own introductory letters and consent forms (Appendices C and D). The teacher will receive her own introductory letter and consent form (Appendices E and F). The school principal will also receive an introductory letter and consent form (Appendices G and H). In order to conceal the identities of the board, school, teacher, and students, names will not be used and all associated wording in appendices will be altered to preserve anonymity.

The unit on fractions will be taught over a period of 3 to 4 weeks. The retention test will be given six weeks after the end of the unit. Generally, the math period will be about 90 minutes in length, and be scheduled in the late morning before the lunch break. The tests and fraction unit teaching plan have been developed to address six areas of student difficulty with fractions identified in the literature (Tables 1 and 2, respectively).

Table 1. Instrument Design

Letter	Difficulty cited in the literature	Citation	Instrument item	
			Pre-test	Mid-test
A	Fractional parts are equal-size portions	Pothier & Sawada (1983); Reys et. al (1999)	1	1
B	Understanding fraction symbols	Mack (1995); Van de Walle (2007)	2	3
C	Improper fractions	Mack (1990); Tzur (1999)	3	3, 8
D	Size of unit fractions	Mack (1990); Van de Walle (2007)	4	4, 9
E	Estimation and comparison: fraction number sense	Cramer & Henry (2002); Cramer et al. (2002); Reys et al. (1999)	5, 6	5, 6
F	Equivalent fractions	Kamii & Clark (1995); Jigyel & Afamasaga-Fuata'i (2007)	7	2, 7

Table 2. Fraction Unit Teaching Plan

Lesson	Topic	Difficulty addressed
Pre-test		all
1	Fractions as parts of sets (Burns, 2001; Van de Walle et al., 2011)	B, E
2	Introduce fair-sharing with children's literature: *The Doorbell Rang* (Hutchins, 1986)	A, B, C, D, F
3	More fair sharing tasks in context, e.g., brownies, candy bars (Burns, 2001; Empson, 2002; Flores & Klein, 2005; Fosnot & Dolk, 2002; Sharp et al., 2002; Van de Walle et al., 2011)	A, B, C, D, F
4	Investigation (Submarine Sandwiches, adapted from Fosnot & Dolk, 2002)	A, B, D, E, F
5	Finish and take up Investigation in a congress (Fosnot & Dolk, 2002)	A, B, D, E, F
6	Make rectangular fraction kits (Burns, 2001)	A, B, D, F
7	Play Cover-up and Uncover games with rectangular fraction kits	A, B, D, F

	(Burns, 2001)	
8	Discuss games, play Uncover with an altered rule, do Cover the Whole (Burns, 2001)	A, B, D, F
9	Drawing fractional parts of sets (Burns, 2001; Van de Walle et al., 2011)	A, B, F
10	Fraction benchmarks (Burns 2001; Reys et al., 1999; Van de Walle et. al 2011)	B, E, F
11	Ordering fractions (Burns, 2001)	B, C, D, E, F
12	Adding and subtracting fractions (Burns, 2003)	B, C, F
Mid-test		all
13	Minilesson: Addition using landmark fractions of an hour, The clock B1 (Imm, Fosnot & Uittenbogaard, 2007); Halving squares (Burns, 2001)	A, B, F
14	Minilesson: Addition using landmark fractions of an hour, The clock B2 (Imm et al., 2007); Exploring fractions with pattern blocks (Burns, 2001)	A, B, D, F
15	Minilesson: Addition using landmark fractions of an hour, The clock B3 (Imm et al., 2007); Wipeout game (Burns, 2001)	A, B, D, F
16	Minilesson: Subtraction using landmark fractions of an hour, The clock B7 (Imm et al., 2007); How much is blue? A pattern block activity (Burns, 2001)	A, B, D, F
17	Minilesson: Subtraction using landmark fractions of an hour, The clock B8 (Imm et al., 2007); Problems with oranges (Burns, 2003)	B, C, E, F
18	Minilesson: Mixed addition and subtraction using landmark fractions of an hour, The clock B9 (Imm et al., 2007); Fraction Capture Game (Burns, 2003)	A, B, D, E, F
19	Minilesson: Addition of fractions using equivalence, The Double Number Line (Imm et al., 2007); Fraction Capture Game, Version 2 (Burns, 2003)	A, B, D, E, F
20	Minilesson: Addition of fractions using equivalence, The Double Number Line (Imm et al., 2007); Balloons and brownies (Burns, 2003)	A, B, C, D
Post-test		all
Retention test	6 weeks after the Post-test	all

The unit will begin with the pre-test (Appendix I). Students will be advised to do their best and that the results will assist in lesson planning for the unit. The pre-test will not be timed; students who require extra time to complete the test will be able to do so during the lunch break that day.

The lessons will be taught according to reform methods of instruction. The lesson plans will be developed in collaboration with the classroom teacher and take into consideration the results of the pre-test as well as recommendations from the literature. Lessons 13 to 20 will be videotaped to capture a record of the instruction. With parental permission, some students will be videotaped as they solve problems together in class to provide a record of the development of their thinking. Observation notes will be recorded for each lesson and will include matters such as how the lesson was taught, students' responses to situations, possible improvements, and direction for the next lesson.

The mid-test (Appendix J) will be administered prior to the introduction of the clock model, in order to discern whether students have already developed the model for themselves and how much.

The unit will end with the post-test, which will be included in student evaluation. Six weeks after the end of the unit, a retention test will be given. The post- and retention tests will be amalgamations of the items from the pre- and mid-tests, with slightly different numbers and contexts. These items will be chosen after examining the results from the pre- and mid-tests and deciding what will be the most fruitful areas to examine. The results of the retention test will not be used for student evaluation, only for the purposes of the study.

Data Collection

Consistent with case study methodology, data collection will draw on multiple sources of information in order to build “an in-depth picture of the case” (Creswell, 2007, p. 132). The main sources of data will be the assessments and videotapes. Some student work completed in class will also be collected. Only data from students who have given consent, and whose parents have given consent, will be used.

Student work will be completed on notepaper and chart paper. Some student work will be collected, photocopied, and returned to the students the following class. At the end of the unit, student duo-tangs will be collected in order to trace the development of their thinking.

While data will be gathered for all who give consent, detailed analysis about the progress of six students will be carried out. Two will be chosen as representative of each part of the spectrum — low, mid, and high achieving students — since “[m]ore research into possible differential effects for high and low achieving pupils is needed” (van Dijk, van Oers, & Terwel, 2003, p. 70). Selection of these students will be made on the recommendation of the teacher on the basis of their general progress in mathematics during this school year as well as the results of the pre-test.

During the period of instruction, some students will be videotaped in order to obtain high quality data on what the students know and can do. In order to minimize disruption to the class, the video camera will be placed on a tripod at the side/back of the room. The remote zoom feature will allow a closer look at how the students are interacting and solving problems together. The tapes will be watched the same evening and notes made.

Data Analysis

Data will be described, classified, interpreted, and represented (Creswell, 2007) in the process described below.

Each student will be assigned a number which will be used to identify his or her work. The students will not know their numbers. The pre-, mid-, post-, and retention tests will be labelled with the student number after the tests are handed in. Student work that is collected will also be labelled with the number after it has been photocopied. I will mark all sets of tests. The responses will be coded either as correct or incorrect and also coded by type of solution strategy, model used, and/or big idea addressed. The categories for coding are based on *The landscape for fractions, decimals, and percents*, developed by C. T. Fosnot and M. Dolk (Fosnot & Dolk, 2002, pp. 136-137). Once everything has been coded, the solutions will be re-examined to investigate evidence of the use of the clock model. It is expected that students will demonstrate the use of the clock model in their solutions on the post- and retention tests, but not on the pre- and mid-tests.

Additional data will be obtained from videotapes taken in class of the students solving the problems as well as in-class work by students and my observation journal. Some in-class work will be collected daily and coded. My observation journal entries will reflect my personal view of the lesson taught that day and observed student reactions to the lessons. These observed reactions will provide me with insight into how the students are making use of the clock model in solving problems.

The purpose of the videotaping is two-fold: first, to answer the research question and second, to have available for later professional development for in-service teachers. It is important to note the teacher's method of introducing the clock model as well as the processes the students use when solving problems. During the screening of the video, students' verbal and written responses will be analyzed. The video segments will be coded with the use of ATLAS.ti, a software program designed to assist researchers when analyzing qualitative data. Each video will be viewed and coded using the same system as described for the written responses. The expectation is that after the clock model has been co-constructed, students will make use of it to solve problems. It is also expected that their use of the clock model will progress from rudimentary to more sophisticated as the unit progresses. A summary of data sources and analysis is portrayed in Table 3.

Table 3. Summary of Data Sources and Analyses

Data source	**Type of analysis**
Pre-, mid-, post-, and retention test responses	Code correct/incorrect and for strategy, model, and/or big idea.
Video recordings	Summarize to analyze role of the teacher and student use of models in solving problems Code for strategy, model, and/or big idea
Photocopied student work	Examples of student use of models for solving problems
Observation journal	Review for role of the teacher and evidence of student use of models

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Appendix A: Parent Letter

(to be printed on letterhead)

February 2011

Dear Parent/Guardian of Potential Participant;

My name is Susan Girardin and I am working on my Master of Education degree at Lakehead University. My goal for my thesis is to investigate an area in mathematics where students have difficulty learning and to find ways to improve the teaching of this topic. The focus of my research is on learning fractions and the use of models to understand fractions concepts. The title of my study is "The role of co-construction of the clock model in the development of fractional understanding."

I will be observing mathematics lessons in your child's classroom during the unit on fractions. The unit will be taught for 4 weeks during February and March 2011. The students will take a pre-test, mid-test, post-test, and retention test (in April) to determine what they have learned in the unit. Some samples of students' work will be collected. During some of the lessons, [teacher]'s teaching methods will be videotaped. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Their conversations may be transcribed and quoted anonymously in my final project in order to illustrate their use of models. I, [teacher], or my supervisor Dr. Lawson, may also make use of some of the edited classroom footage and work samples for professional development for teachers. Upon completion of the project, you will be welcome to obtain a summary of the research by contacting me at the phone number or email address given below, or by giving your mailing address or email address on the consent form.

Your child will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If video data is used for professional development, your child will be identified by first name only. The raw data that is collected will be securely stored at Lakehead University for five years and then destroyed. Participation in this study is voluntary and you may withdraw the use of your child's data at any time, for any reason, without penalty. The research project has been approved by the Lakehead University Research Ethics Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 343-8283 or swright@lakeheadu.ca. The research has been approved by the Lakehead District School Board and the Principal of [Name of] School.

Please note that this research does not affect classroom instruction time, with the exception of the 45-minute retention test. The lessons are being carried out by [teacher] in the same manner and length of time as they would be without the research project. This research will not take away from the normal learning environment in the classroom and there is no apparent risk to your child. The research is simply being conducted to make note of the effects of using a model, which is a regular part of the fractions unit. If you choose not to have your child participate, he or she will still be engaged in the math lessons. The only difference is that his or her data will not be used. If you give permission for your child to participate, your child will also be asked whether he or she is willing to take part in this research.

You are welcome to contact me at 344-3465 or sgirardi@lakeheadu.ca if you have any questions concerning this research project. I would be very pleased to speak with you.

If you agree to allow your child to participate in the study, please sign the attached letter of consent and return it to [teacher] at the school. Please keep this letter in case you would like to contact any one of us.

Sincerely,

Susan Girardin

Mrs. S. Girardin Master's Student Lakehead University 807-344-3465 sgirardi@lakeheadu.ca	Dr. A. Lawson, Ph.D. Thesis Supervisor Lakehead University 807-343-8720 alawson@lakeheadu.ca
Principal [Name of] School 807-	Sue Wright Research Ethics Board Lakehead University 807-343-8283 swright@lakeheadu.ca

Appendix B: Parent Consent Form

(to be printed on letterhead)

Parent Consent Form

I DO give permission for my son/daughter, ____,
(Student's Name/please print)

to participate in the study with Susan Girardin as described in the attached letter.

I understand that:

1. My child will be videotaped in the classroom environment as part of the research.
2. My child's participation is entirely voluntary and I can withdraw permission at any time, for any reason, with no penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then destroyed.
5. All participants will remain anonymous in any publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by me, [teacher], or Dr. Lawson. If my child appears in the video clips he/she will be identified only by first name.

☐ I initial this box to give permission for my child to appear in video clips which may be used for Professional Development purposes, as outlined in 6. above.

7. I can receive a summary of the project, upon request, following the completion of the project, by calling or writing, or by providing my address or email address below.

Please keep the introductory letter on file should you have any further questions.
If you agree to let your child take part in the study, please complete this page and have your child return it to [teacher].

____
Name of Parent/Guardian (please print)

____ ____________________
Signature of Parent/Guardian Date

Address or email address (if you would like a summary of the findings):

____

Appendix C: Potential Participant Letter

(to be printed on letterhead)

February 2011

Dear Potential Participant;

In February and March I will be coming to your classroom while [teacher] teaches fractions. I will be paying attention and writing things down during your math classes because I am curious about what helps people to learn fractions best. I am a student at Lakehead University in the Master of Education program and this is part of my school project, “The role of co-construction of the clock model in the development of fractional understanding”.

[Teacher] will teach the lessons to you just like s/he usually does. A difference you will notice is that during some of lessons there will be a video camera in the classroom and a microphone on your work table. These tools will help me with my project by recording what you say and do while you are solving problems. I, [teacher], or my supervisor Dr. Lawson, may also want to use some video clips from the classroom and samples of your work for helping other teachers learn more about how to teach about fractions. If you are in a video that will be seen by other teachers, your first name might be used. I will not use your name in anything I write about the project.

The unit will start with a pre-test so that I can see what you know about fractions before any of the lessons. [Teacher] will teach the lessons and sometimes your work will be collected. I will photocopy some of the work that [teacher] collects and use it to help me understand your thinking. There will be a test in the middle of the unit to see how you are doing so far, and a test at the end to see what you have learned. Later in April there will be another test, called a retention test, to see what you remember.

Please ask me any questions you have about my project and I will be happy to answer them. You can decide whether or not to be part of my project. You will be doing the same work in math class whether you are in my project or not, the only difference is that I will not use your test results or your work or any video clips with you in them if you decide not to take part. Thank you for thinking about being part of my project.

Sincerely,

Mrs. Susan Girardin

Appendix D: Potential Participant Consent Form
(to be printed on letterhead)

Potential Participant Consent Form

I, ____, want to take part in the project with
(Student's Name/please print)

Mrs. Susan Girardin as described in the letter.

I understand that:

1. I will be videotaped in the classroom as part of the project.

2. I don't have to take part in the project, but I want to be part of it, and I know I can change my mind about that later and it wouldn't be a problem.

3. It is safe to be part of this project.

4. All of the information Mrs. Girardin collects for her project will be kept in a very safe place at Lakehead University for five years and then it will be destroyed.

5. My name will never be used in anything Mrs. Girardin writes about the project.

6. Mrs. Girardin, [teacher], or Dr. Lawson might want to use some of the videos or copies of my work to help other teachers learn about teaching fractions. My first name might be used in video clips of the classroom. My name will not be on any copies of my work.

☐ I put my initials in this box to show that it is alright for me to appear in video clips which may be used for helping other teachers learn about teaching fractions.

If you want to be part of my project, please fill in this page and give it to [teacher].

____
Name of Student (please print)

____ ________________
Signature of Student Date

Appendix E: Teacher Letter

(to be printed on letterhead)

February 2011

Dear [teacher],

Thank you for considering participation in this study. My goal for my master's thesis is to investigate an area in mathematics where students often have difficulty learning and to find ways to improve the teaching of this topic. The focus of my research is on learning fractions and the use of models to understand fractions concepts. This study is designed to explore the impact on students' thinking when a teacher and students develop the clock model together in the context of a unit on fractions. The title of my study is "The role of co-construction of the clock model in the development of fractional understanding." Presently there is very little information available about the effects of the co-construction of the clock model.

In order to gather the information needed for the study, I will be observing mathematics lessons in your classroom during the unit on fractions. The students will take a pre-test, mid-test, post-test, and retention test to determine what they have learned in the unit. You will have access to the test results for student assessment. Some samples of students' work will be collected. During some of the lessons, your teaching methods will be videotaped. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Conversations may be transcribed and quoted anonymously in my final project in order to illustrate their use of models. You, I, or my supervisor Dr. Lawson, may also make use of some of the edited classroom footage and student work samples for professional development for teachers.

As part of the project you will need to: distribute and collect the cover letters and permission forms from parents or guardians and students; collect student work; and, allow time for the tests (including the retention test). I will ensure that you have any of the resources you might need for the lessons. I hope that you will participate for the duration of the study; however, you may withdraw at any time, for any reason, without penalty, as your participation is entirely voluntary. I do not anticipate any negative consequences as a result of participation in this study.

You and your students will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If video data is used for professional development, your students will be identified by first name only, but if children use your surname it may be revealed. The raw data that is collected will be securely stored at Lakehead University for five years after completion of the project and then destroyed. A report of the research will be available upon request. I can be reached at 344-3465 or sgirardi@lakeheadu.ca.

The research project has been approved by the Lakehead University Research Ethics Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 343-8283 or swright@lakeheadu.ca.

If you agree to participate in the study, please sign the attached letter of consent and return it to me.

Sincerely,

Susan Girardin

Mrs. S. Girardin
Master's Student
Lakehead University
807-344-3465
sgirardi@lakeheadu.ca

Dr. A. Lawson, Ph.D.
Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
swright@lakeheadu.ca

Appendix F: Teacher Consent Form
(to be printed on letterhead)

Teacher Consent Form

I ____, do agree to participate in the study with
(Teacher's Name/please print)

Susan Girardin as described in the attached letter.

I understand that:

1. I will be videotaped in the classroom as part of the research.

2. My participation is entirely voluntary and I can withdraw permission at any time, for any reason, without penalty.

3. There is no apparent danger of physical or psychological harm.

4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then destroyed.

5. I will remain anonymous in any publication resulting from the research project.

6. The video clips of the classroom or my work may be included in Professional Development for teachers conducted by Susan Girardin, myself, or Dr. Lawson. If I appear in the video clips I may be identified by surname.

☐ I initial this box to give permission for me to appear in video clips which may be used for Professional Development purposes, as outlined in 6. above.

____
Name of Third Party Witness (please print)

____ _______________
Signature of Third Party Witness Date

If you agree to take part in my study, please complete this page and return it to me.

____
Name of Teacher (please print)

____ _______________
Signature of Teacher Date

Appendix G: Principal Letter

(to be printed on letterhead)

February 2011

Dear [Principal's Name],

Thank you for considering participation in this study. My goal for my master's thesis in education is to investigate an area in mathematics where students often have difficulty learning and to find ways to improve the teaching of this topic. The focus of my research is on learning fractions and the use of models to understand fractions concepts. This study is designed to explore the impact on students' thinking when a teacher and students develop the clock model together in the context of a unit on fractions. The title of my study is "The role of co-construction of the clock model in the development of fractional understanding." Presently there is very little information available about the effects of the co-construction of the clock model.

In order to gather the information needed for the study, I will be observing mathematics lessons in [teacher]'s classroom during the unit on fractions. The students will take a pre-test, mid-test, post-test, and retention test to determine what they have learned in the unit. She will have access to the test results for student assessment. Some samples of students' work will be collected. During some of the lessons, [teacher]'s teaching methods will be videotaped. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Conversations may be transcribed and quoted anonymously in my final project in order to illustrate their use of models. I, [teacher], or my supervisor Dr. Lawson, may also make use of some of the edited classroom footage and student work samples for professional development for teachers.

This research does not affect classroom instruction time, with the exception of the 45-minute retention test. The lessons are being carried out by [teacher] in the same manner and length of time as they would be without the research project. This research will not take away from the normal learning environment in the classroom and there is no apparent risk. If parents choose not to have a child participate, the child will still be engaged in the math lessons. The only difference is that his or her data will not be used. If parents give permission for a child to participate, the child will also be asked whether he or she is willing to take part in this research.

I hope that [teacher] and her students will participate for the duration of the study; however, you may withdraw your permission at any time, for any reason, without penalty, as participation is entirely voluntary. I do not anticipate any negative consequences as a result of participation in this study.

The School Board, [name of] School, [teacher], and her students will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If video data is used for professional development, the students will be identified by first name only; however, if students use the teacher's surname it may be revealed. The raw data that is collected will be securely stored at Lakehead University for five years after completion of the project. A report of the research will be available upon request. I can be reached at 344-3465 or sgirardi@lakeheadu.ca.

The research project has been approved by the Lakehead University Research Ethics Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 343-8283 or swright@lakeheadu.ca.

If you give permission for participation in the study, please sign the attached letter of consent and return it to me.

Sincerely,

Susan Girardin

Mrs. S. Girardin
Master's Student
Lakehead University
807-344-3465
sgirardi@lakeheadu.ca

Dr. A. Lawson, Ph.D.
Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
swright@lakeheadu.ca

Appendix H: Principal Consent Form
(to be printed on letterhead)

Principal Consent Form

I ______________________________________, do agree to participation in the study with
(Principal's Name/please print)

Susan Girardin as described in the attached letter.

I understand that:

1. [teacher] and her students will be videotaped in the classroom as part of the research.

2. Their participation is entirely voluntary and I can withdraw permission at any time, for any reason, without penalty.

3. There is no apparent danger of physical or psychological harm.

4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then destroyed.

5. The Lakehead District School Board, [name of] School, [teacher], and her students will remain anonymous in any written publication resulting from the research project.

6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by Susan Girardin, [teacher], or Dr. Lawson. If students appear in the video clips, they will only be identified by first name. If [teacher] appears in the video clips, she may be identified by surname.

☐ I initial this box to give permission for [teacher] and her students to appear in video clips which may be used for Professional Development purposes, as outlined in 6. above.

If you approve of participation in my study, please complete this page and return it to me.

____
Name of Principal (please print)

______________________________________ ____________
Signature of Principal Date

Appendix I: Pre-Test

Fractions Pre-Test

1. 4 children are sharing 10 brownies so that everyone gets the same amount.
   How much brownie can 1 person have? (Empson, 2002, pp. 29-30)

2. What does the bottom number in a fraction tell us?

   What does the top number in a fraction tell us? (Van de Walle, 2007, p. 299)

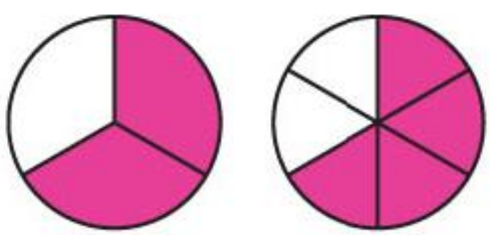
3. If Linda ate one half of an apple pie and two thirds of a cherry pie, how much did she eat? (adapted from Sharp et al., 2002, p. 21)

4. Put these fractions in order from smallest to largest: 1/5, 1/3, and ¼
   (Cramer & Henry, 2002, p. 43)

5. Joey and Robert each had the same size pizza. Joey cut his pizza into eight equal pieces and ate six of them. Robert cut his into five equal pieces and ate four of them.
   Who ate more pizza? (Burns, 2001, p. 138)

6. Is the following sum larger or smaller than 1? Explain how you know. Use estimation.
   ½ + 1/3 (Reys et al., 1999, p. 530)

7. Are the following shaded circles equal? Compare and explain your answer.
   (Jigyel & Afamasaga-Fuata'i, 2007, p. 21)



8. Add. ¾ + 2/12 (Imm et al., 2007, p. 24)

9. Joel worked out at the Complex the other day. He ran for half of an hour and then walked for ¼ of an hour. How much of the hour did he spend exercising?
   (adapted from Imm et al., 2007, p. 27)

Appendix J: Mid-Test

Fractions Mid-Test

1. There are 4 kids at a party and there are 7 brownies for them to share.
   How can they share them? (Sharp et al., 2002 p. 20)

2. Does 4/6 = 2/3? How do you know?
   (adapted from Van de Walle et al., 2011, p. 310)

3. Subtract: 4 – 7/8 (Mack, 1990, p. 24)

4. Which fraction is bigger, 1/6 or 1/8? Explain your answer. (Mack, 1990, p. 22)

5. Which is larger, 6/8 or 4/5? Explain your reasoning. (Burns, 2001, p. 137)

6. Is the following sum larger or smaller than 1? Explain how you know. Use estimation.
   3/8 + 4/9 (Reys et al., 1999, p. 530)

7. At her birthday party, Janie blew out ¾ of the candles on her cake. Draw a picture that shows the birthday cake and candles, also showing which candles were blown out.
   P.S. How old is Janie? Give two ages that Janie might be. Explain your reasoning.
   (adapted from Burns, 2001, p. 142)

8. Suppose you have four cookies and you eat 7/8 of one cookie, how many cookies do you have left? (Mack, 1990, p. 24)

9. Suppose you have two pizzas of the same size, and you cut one of them into six equal-sized pieces and you cut the other one into eight equal-sized pieces.
   If you get one piece from each pizza, which one do you get more from?
   (Mack, 1990, p. 21)

10. Add. 2/3 + 1/6 (Imm et al., 2007, p. 25)

11. Joel exercised at the Complex one day last week. He ran really fast for 1/6 of an hour and then ran at his normal pace for ½ an hour. How much of the hour did he spend running? (adapted from Imm et al., 2007, p. 24)