Taper Equations for Five Major Commercial Tree Species in Manitoba, Canada

Ryan J. Klos, G. Geoff Wang, Qing-Lai Dang, and Ed W. East

ABSTRACT

Kozak's variable exponent taper equation was fitted for balsam poplar (*Populus balsamifera L.*), trembling aspen (*Populus tremuloides* Michx.), white spruce (*Picea glauca* [Moench] Voss), black spruce (*Picea mariana* [Mill.] B.S.P.), and jack pine (*Pinus banksiana* Lamb.) in Manitoba. Stem taper variability between two ecozones (i.e., Boreal Shield and Boreal Plains) were tested using the F-test. Regional differences were observed for trembling aspen, white spruce, and jack pine, and for those species, separate ecozone-specific taper equations were developed. However, the gross total volume estimates using the ecozone-specific equations were different from those of the provincial equations by only 2 percent. Although the regional difference in stem form was marginal within a province, a difference of approximately 7 percent of gross total volume estimation was found when our provincial taper equations were compared with those developed in Alberta and Saskatchewan. These results suggest that stem form variation increases with spatial scale and that a single taper equation for each species may be sufficient for each province.

Keywords: Diameter inside bark, ecozone, gross total volume, growth and yield, stem form

Balsam poplar (*Populus balsamifera* L.), trembling aspen (*Populus tremuloides* Michx.), white spruce (*Picea glauca* [Moench] Voss), black spruce (*Picea mariana* [Mill] B.S.P.), and jack pine (*Pinus banksiana* Lamb.) are major commercial tree species in Manitoba. Forest ecosystems containing these tree species occur over a large geographical area, with varying climate, site conditions, and forest productivity. The ability to predict the growth and yield of forest stands located in various climate and site conditions is critical in the development of ecologically based management plans and strategies. However, information regarding growth and yield and relationships between forest productivity and climate and site variables is currently lacking in Manitoba.

Among the essential building blocks in forest growth and yield modeling are the equations/models for estimating individual tree volume of different species. The use of taper equations in estimating individual tree volume has recently become an increasingly popular trend (Huang 1994). Taper equations have been shown to provide accurate diameter inside bark (diameter inside bark) predictions in Canada (e.g., LeMay 1982, Gal and Bella 1994, Huang 1994). In particular, Kozak's taper equation (Kozak 1988) has been proven to fit well to many tree species (Perez et al. 1990, Kozak 1991, LeMay et al. 1993). However, taper equations for major tree species have not been developed in Manitoba. The current tree volume estimation procedures used in Manitoba are not based on taper equations and do not consider ecological differences in climate and site conditions. Previous testing of individual tree volume equations for black spruce has revealed significant bias and error (Wang 1997).

Variations in stem taper for different trees are the result of differences in diameter and height growth along the stem over time (Muhairwe 1999). Therefore, factors that affect tree growth in height and diameter (e.g., genetics, climatic fluctuations, site quality, tree and stand age, crown size, canopy position, defoliation, species, and stand density) also affect taper (Muhairwe 1994). For two trees with comparable sizes (in terms of total tree height and dbh), the tree with less taper can have as much as 20 percent more volume (Heger 1965). To account for regional variations in stem taper, Huang (1994) used Kozak's model to develop regionalized taper equations for Alberta natural subregions. Regional differences in stem taper were found to be statistically significant, likely due to the differences in biological, geographical, and climate conditions (Huang et al. 2000). These regionalized equations performed well at different portions of the stem and for various tree sizes (Huang et al. 2000). Similarly, Wang and Huang (2000) and Wang et al. (2004) also found that the height growth patterns of white spruce and lodgepole pine were different among ecozones in Alberta and that ecozone-based height growth curves, instead of a single provincial curve, were recommended.

The objective of this study was to develop ecologically based taper equations for major commercial tree species in Manitoba using Kozak's model. Specifically, we tested stem taper variability between ecozones; on the basis of this variability, regionalized taper equations were constructed if needed. The species of interest include balsam poplar, trembling aspen, white spruce, black spruce, and jack pine.

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Ryan J. Klos, Faculty of Forestry and the Forest Environment, Lakehead University, 955 Oliver Rd., Thunder Bay, Ontario P7B 5E1, Canada. G. Geoff Wang can be reached at gwang@clemson.edu. Department of Forestry and Natural Resources, Clemson University, 261 Lehotsky Hall, Clemson, South Carolina, 29634-0317. Qing-Lai Dang, Faculty of Forestry and the Forest Environment, Lakehead University, 955 Oliver Rd., Thunder Bay, Ontario P7B 5E1, Canada; Ed W. East, Forestry Branch, Manitoba Conservation, Box 70, 200 Saulteaux Cr., Winnipeg, Manitoba R3J 3W3, Canada. *This study was funded by the Forestry Branch of Manitoba Conservation, LP Canada, Ltd., Swan River Division, and the National Science and Engineering Research Council of Canada. Special thanks go to the Forestry Branch of Manitoba Conservation and LP Canada, Ltd., Swan River Division, for providing the data used in this study. We also thank Dr. Jian R. wang for helpful comments.*

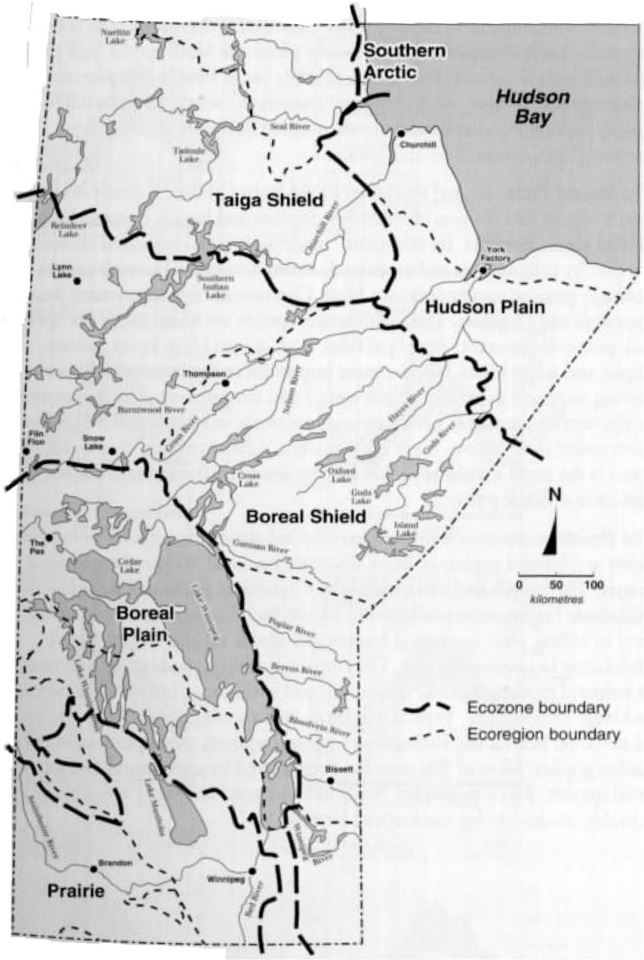

Figure 1. Map of ecozones in the province of Manitoba (Zoladeski et al. 1995).

Materials and Methods

Study Area

The study area covers two ecozones in Manitoba: Boreal Shield and Boreal Plains (Figure 1). The Boreal Shield ecozone is characterized by long, cold winters and short, warm summers, with mean annual, summer, and winter temperatures ranging between −4° C and 1.5° C, 11.5° C and 15° C, and −20° C and −13° C, respectively (Environment Canada 2004b). The mean annual precipitation ranges between 400 and 700 mm (Environment Canada 2004b). The average annual growing degree-days greater than 5° C is approximately 1,350 (Environment Canada 2004a). This ecozone is dominated by broadly rolling uplands and lowlands with many bedrock outcrops and small to medium lakes (Zoladeski et al. 1995). Luvisol soils are dominant in the southern portion, whereas Brunisols are dominant in the northern portion (Zoladeski et al. 1995). The Boreal Plains ecozone is characterized by cold winters and moderately warm summers, with mean annual, summer, and winter temperatures ranging between −1° C and 1° C, 13° C and 15.5° C, and −17° C and −13.5° C, respectively (Environment Canada 2004b). The mean annual precipitation ranges between 375 and 625 mm (Environment Canada 2004b). The average annual growing degree-days greater than 5° C is approximately 1,550 (Environment Canada 2004a). The ecozone is covered with a relatively flat to gently rolling landscape consisting of lacustrine deposits and large hummocky to kettled glacial moraine (Zoladeski et al. 1995). Luvisol soils are dominant across the ecozone; however, Black Chernozems exist in the southern portion and Brunisols and Organic soils exist in the northern portion (Zoladeski et al. 1995).

Data Collection

The stem analysis data used in this study were collected by the Forestry Branch of Manitoba Conservation. LP Canada, Ltd., Swan River division, assisted with the collection of the trembling aspen and balsam poplar data in the Boreal Plains ecozone.

Stands were selected on the basis of the following criteria: (1) pure, (2) fully stocked, (3) even-aged, and (4) minimal or no disturbances. Plots were avoided near major roads because road construction might have altered soil drainage and affected tree growth. Once a stand was located, either a 300-m^2 circular plot or a 625-m^2 square plot was established. Three trees in the dominant and codominant crown classes were selected, representing the largest, smallest, and average dbh outside bark (dbhob). These trees were felled, and disks (cross-sectional slices of the stem) were obtained from 0.33 m, 0.67 m, 1.0 m, 1.3 m, and every 1.3-m interval thereafter until a diameter outside bark (dob) of 7.0 cm was reached. Disks were then taken every 20 cm between 7.0 and 4.0 cm dob. Many variables were recorded for each tree; however, for the purpose of this study, only diameter inside bark (dib), height above ground, dbhob, and total tree height were used. A summary of the dbhob and total tree height data is shown in Table 1. Note that balsam poplar was not sampled and that only 10 trembling aspen were sampled in the Boreal Shield ecozone.

Data Analysis

On the basis of previous studies conducted in two other prairie provinces (i.e., Alberta and Saskatchewan) (Huang 1994, Gal and Bella 1994), Kozak's variable exponent model (Kozak 1988) was selected for our study:

$$\mathrm{dib}_i = a_0 D^{a_1} a_2^{D} \cdot X_i \exp\left[b_1 z_i^2 + b_2 \ln(z_i + 0.001) + b_3\sqrt{z_i} + b_4 e^{z_i} + b_5(D/H)\right], \quad (1)$$

with

$$X_i = (1 - \sqrt{h_i/H})/(1 - \sqrt{p}),$$

where dib_i is diameter inside bark (cm) at point i along the stem; D is dbhob (cm); h_i is height above ground (m) at point i along the stem; H is total tree height (m); p is relative height constraint (p = 0.25); z_i is relative height (h_i/H); e = 2.71842; and a_0, a_1, a_2, b_1, b_2, b_3, b_4, and b_5 are parameters to be estimated. Muhairwe (1999) suggested that using an average value of 0.25 for p is appropriate.

Equation 1 poses two potential problems: multicollinearity and autocorrelation. Multicollinearity is commonly found in overcomplicated equations with several polynomial terms. The problem of autocorrelation arises from using multiple observations from the same tree. However, the estimates are still unbiased, and accounting for these problems is of little importance when the best predictions are desired (Kozak 1997, Huang et al. 1997).

To facilitate fast convergence (Huang 1994), initial values of the parameters in Equation 1 were estimated on the basis of its linearized form, as follows.

$$\ln(\mathrm{dib}_i) = \ln(a_0) + a_1\ln(D) + \ln(a_2)\,D + b_1\ln(X_i)\,z_i^2$$

Table 1. **Summary statistics of the stem analysis data used in this study.**

Species		No. of trees	Variable	Mean	Min[a]	Max	SD
Balsam poplar	Boreal Plains ecozone	37	D (cm)	24.3	17.9	31.0	3.1
			H (m)	21.03	16.90	24.50	2.23
Trembling aspen	Boreal Shield ecozone	10	D (cm)	19.9	9.7	29.3	6.0
			H (m)	19.61	9.90	28.10	5.13
	Boreal Plains ecozone	61	D (cm)	24.1	11.5	35.1	5.1
			H (m)	21.43	11.70	26.40	3.08
	Provincial	71	D (cm)	23.5	9.7	35.1	5.4
			H (m)	21.20	9.90	28.10	3.47
White spruce	Boreal Shield ecozone	33	D (cm)	23.5	13.3	37.1	6.7
			H (m)	18.17	11.56	25.65	3.68
	Boreal Plains ecozone	64	D (cm)	27.9	14.7	40.4	7.1
			H (m)	21.23	11.84	29.30	4.52
	Provincial	97	D (cm)	26.4	13.3	40.4	7.3
			H (m)	20.22	11.56	29.30	4.50
Black spruce	Boreal Shield ecozone	202	D (cm)	16.6	5.5	25.9	3.8
			H (m)	15.36	5.96	25.00	3.41
	Boreal Plains ecozone	98	D (cm)	15.4	5.3	25.8	4.5
			H (m)	15.04	4.66	23.70	4.26
	Provincial	300	D (cm)	16.2	5.3	25.9	4.0
			H (m)	15.26	4.66	25.00	3.70
Jack pine	Boreal Shield ecozone	168	D (cm)	17.5	2.9	31.2	6.1
			H (m)	14.57	3.30	24.60	4.59
	Boreal Plains ecozone	130	D (cm)	18.0	4.7	31.2	5.2
			H (m)	14.93	5.50	22.40	4.06
	Provincial	298	D (cm)	17.7	2.9	31.2	5.8
			H (m)	14.73	3.30	24.60	4.36

[a] D, dbh outside bark; H, total tree height; Min, minimum; Max, maximum.

$$+ b_2\ln(X_i)\ln(Z_i + 0.001) + b_3\ln(X_i)\sqrt{z_i} + b_4\ln(X_i)e^{z_i} + b_5\ln(X_i)(D/H). \quad (2)$$

The linear model (Equation 2) was fitted for each species, and the parameter estimates were then used as initial values in the model fitting with the nonlinear procedure.

The nonlinear regression procedure PROC NLIN in the SAS/STAT software (SAS institute, Inc., 1990), based on the Gauss-Newton iterative method (Gallant 1987), was used to estimate the parameters of Equation 1 (Huang 1994). A provincial taper equation was fitted for each species on the basis of all the data, and the mean square error (MSE) and coefficient of determination (R^2) for each fitted equation were then calculated using Equations 3 and 4, respectively:

$$\mathrm{MSE} = \frac{\sum_{i=1}^{n}(\mathrm{dib}_i - \widehat{\mathrm{dib}}_i)^2}{n - m}, \quad (3)$$

where $\widehat{\mathrm{dib}}_i$ is predicted dib (cm) at point i along the stem; n = number of observations; and m is the number of parameters ($m = 8$).

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\mathrm{dib}_i - \widehat{\mathrm{dib}}_i)^2}{\sum_{i=1}^{n}(\mathrm{dib}_i - \overline{\mathrm{dib}})^2}, \quad (4)$$

where $\overline{\mathrm{dib}}_i$ is observed average dib (cm).

The nonlinear extra sum of squares procedure as demonstrated by Bates and Watts (1988) and Huang (1994) was used to determine whether differences of stem taper existed between ecozones. Since Equation 1 possesses eight parameters, using indicator variables for all the parameters would overparameterize the equation. Indicator variables are variables that account for categorical differences of parameters in regression equations. Indicator variables are frequently applied to models that allow for behavioral differences in geographic regions (Judge et al. 1988, Neter et al. 1990). The parameters a_1 and b_5 were the only parameters represented by indicator variables, since they were highly correlated with dib and were used by Huang (1994) and Huang et al. (2000). Using indicator variables only for the highly correlated parameters is common for models with three or more parameters (Bates and Watts 1988). The full model (Equation 5), which accounts for ecozone differences, can be expanded from the provincial Equation 1, the reduced model:

Table 2. **Fit statistics of the provincial taper equations developed by fitting Equation 1 to the provincial data of each species.**

Parameter[a]	Balsam poplar	Trembling aspen	White spruce	Black spruce	Jack pine
a_0	0.2569	0.6549	0.6977	0.8894	0.9674
a_1	1.5383	1.1280	1.1086	1.0163	0.9746
a_2	0.9751	0.9930	0.9931	0.9950	0.9976
b_1	0.7719	0.9223	0.2342	0.2866	0.1459
b_2	−0.1105	−0.1311	−0.0653	−0.0853	−0.0402
b_3	0.1448	0.6458	0.1814	0.6307	0.0487
b_4	−0.0811	−0.2967	0.0491	−0.1714	0.1266
b_5	0.1324	0.0670	0.1233	0.1491	0.0753
MSE	1.236146	1.768216	1.769892	0.663633	1.244656
R^2	0.983187	0.976623	0.983337	0.981481	0.974126
n	875	1624	2160	5940	5430

[a] a_0, a_1, a_2, b_1, b_2, b_3, b_4, and b_5, parameter estimates; MSE, mean square error.

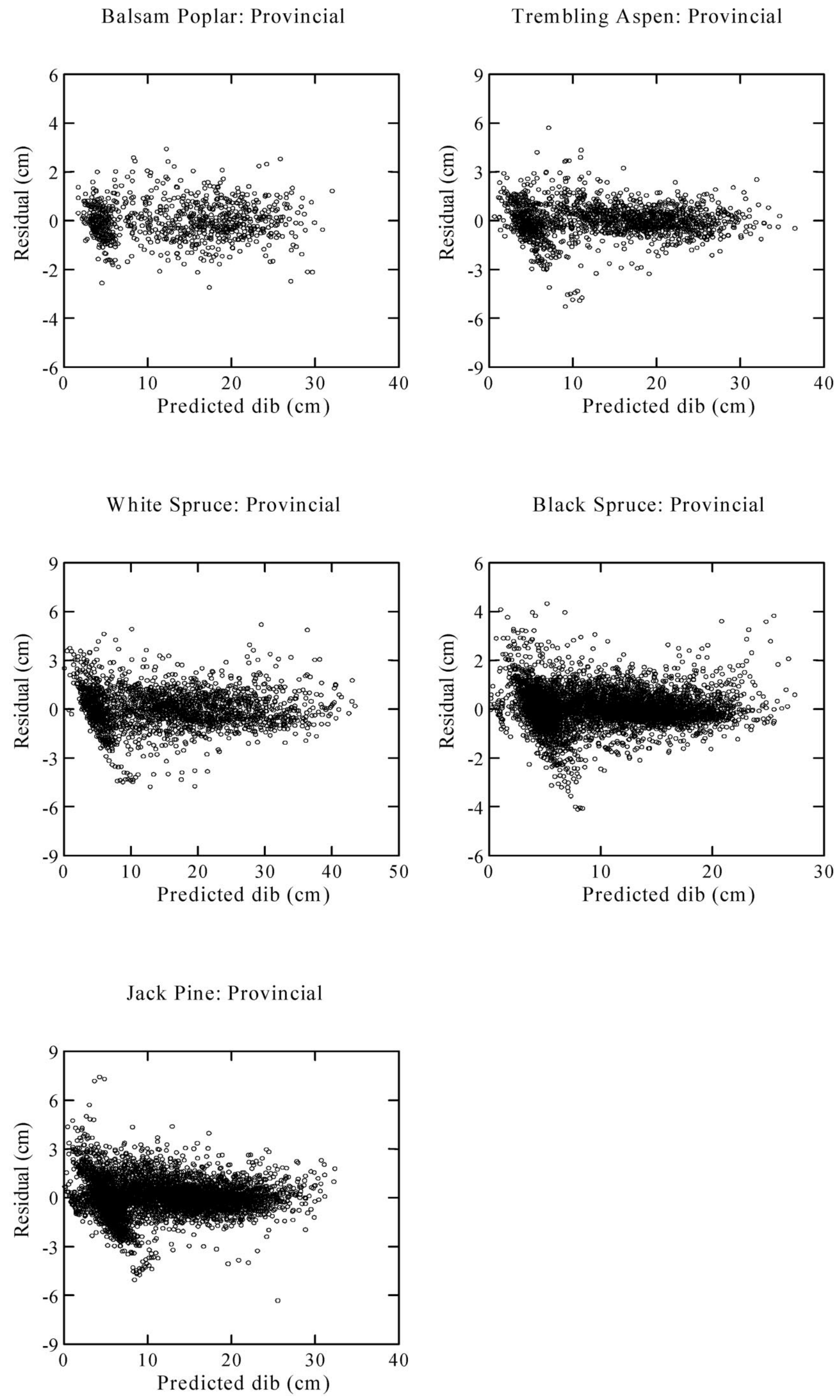

Figure 2. Residual plots of the provincial taper equations for balsam poplar, trembling aspen, white spruce, black spruce, and jack pine.

$$\text{dib}_i = a_0 D^{(a_1 + c_1 k)} a_2^{D} \cdot X_i \exp[b_1 z_i^2 + b_2 \ln(z_i + 0.001) + b_3 \sqrt{z_i} + b_4 e_i^{z} + (b_5 + c_2 k)(D/H)], \quad (5)$$

where k is an indicator variable for ecozone (i.e., $k = 0$ for Boreal Shield ecozone and $k = 1$ for Boreal Plains ecozone); and c_1 and c_2 are parameter estimates for ecozone differences.

Ecozone differences were tested using the null hypothesis (H_0: $c_1 = c_2 = 0$) and the alternative hypothesis (H_1: at least one of the equalities is not true). To determine these differences, the F-test was used and is illustrated in Equation 6:

$$F = \frac{(SSE_R - SSE_F)/(df_R - df_F)}{SSE_F/df_F}, \quad (6)$$

where SSE_R is the error sum of squares associated with the reduced model (Equation 1), SSE_F is the error sum of squares associated with the full model (Equation 5), df_R is the degrees of freedom associated with the reduced model, and df_F is the degrees of freedom associated with the full model. An α-level of 0.05 was specified. If $F > F_{critical}(\alpha, df_R - df_F, df_F)$, reject H_0. Equation 1 was then fitted for each ecozone, and the mean square error (MSE) and coefficient of determination (R^2) of each fitted equation were then calculated using Equations 3 and 4, respectively.

Table 3. F-test for ecozone differences in the taper equation for trembling aspen, white spruce, black spruce, and jack pine.

Species	Reduced model SSE[a]	Reduced model df	Full model SSE	Full model df	F-value	F(critical)
Trembling aspen	2857.4	1616	2782.9	1614	21.60[b]	3.00
White spruce	3808.8	2152	3753.7	2150	15.78[b]	3.00
Black spruce	3936.7	5932	3934.1	5930	1.96	3.00
Jack pine	6748.5	5422	6603.8	5420	59.38[b]	3.00

[a] SSE, error sum of squares; df, error degrees of freedom.
[b] Significant at $\alpha = 0.05$.

Based on taper equations, gross total volume (GTV) estimates were calculated using Newton's formula and procedures illustrated in Huang (1994). GTV was calculated to illustrate the differences in stem form between ecozones in Manitoba and among the three prairie provinces (Alberta, Saskatchewan, and Manitoba). We used the provincial equations developed by Huang (1994) and Gal and Bella (1994) for Alberta and Saskatchewan, respectively. For each species, GTV was calculated for three tree sizes (small, medium, and large). These trees were assigned arbitrary dbh and total height values within the bounds of the data used to construct the regionalized taper equations. Therefore, the sizes were different for each species. The percent differences for the GTV estimates were then calculated relative to the Manitoba provincial GTV estimates using Equation 7:

$$\text{diff} = \frac{\text{GTV}_b - \text{GTV}_a}{\text{GTV}_a} * 100\%, \qquad (7)$$

where GTV_a is the GTV estimate derived from the Manitoba provincial taper equation, and GTV_b is the GTV estimate derived from the ecozone-specific, Alberta, or Saskatchewan taper equations.

All statistical analyses were conducted using SAS/STAT software (SAS Institute, Inc., 1990). Graphs were constructed using SYSTAT software (Wilkinson 1999).

Results

The fit statistics and residual plots of the provincial taper equation for each species are shown in Table 2 and Figure 2, respectively. Based on the fit statistics and residual plots, each taper equation provided a strong fit to the data. The provincial equations explained more than 97 percent of the total variation of dib for each species, indicating a good agreement between observed and predicted dib. Most residuals clustered around 0 (Figure 2), indicating that the model was not biased.

Regional differences were explored for trembling aspen, white spruce, black spruce, and jack pine. The results of the F-tests showed significant differences between the two ecozones for trembling aspen, white spruce, and jack pine, but not for black spruce (Table 3).

The fit statistics and residual plots of the ecozone-specific taper equations for trembling aspen, white spruce, and jack pine are shown in Table 4 and Figure 3, respectively. Based on the fit statistics and residual plots, the ecozone-specific taper equations also provided a strong fit to the data. They explained more than 97 percent of the total variation of dib for each species, and most residuals clustered around 0 (Figure 3).

A comparison of GTV estimates derived from the ecozone-specific and provincial taper equations for trembling aspen, white spruce, and jack pine is given in Table 5. For a given species of the same tree size, the GTV estimates obtained from these equations were quite similar. Regardless of species, the Boreal Shield GTV estimates were, on average, less than the Boreal Plains GTV estimates. The percent difference between the Boreal Shield ecozone and the provincial GTV estimates ranged between −4.0 percent and 1.4 percent, with an average of −1.7 percent. The percent difference between the Boreal Plains ecozone and the provincial GTV estimates ranged between −3.4 percent and 5.7 percent, with an average of 1.5 percent.

A comparison of GTV estimates derived from the provincial taper equations developed for Alberta (Huang 1994), Saskatchewan (Gal and Bella 1994), and Manitoba (this study) is given in Table 6. Differences in the GTV estimates among provinces were larger than those between ecozones in Manitoba. The percent difference in GTV estimates between Alberta and Manitoba ranged between −17.0 percent and −1.3 percent, with an average of −7.4 percent. The percent difference in GTV estimates between Saskatchewan and Manitoba ranged between −10.0 percent and 14.8 percent, with an average of 6.5 percent.

Discussion

Overall, Kozak's variable exponent taper equation fitted our data very well. Previously, the equation has been shown to provide an adequate fit for several species in western provinces (e.g., Huang 1994, Muhairwe et al. 1994, Gal and Bella 1994), and their fit

Table 4. Fit statistics of the ecozone-specific taper equations for trembling aspen, white spruce, and jack pine.

Parameter[a]	Boreal Shield Ecozone: Trembling aspen	Boreal Shield Ecozone: White spruce	Boreal Shield Ecozone: Jack pine	Boreal Plains Ecozone: Trembling aspen	Boreal Plains Ecozone: White spruce	Boreal Plains Ecozone: Jack pine
a_0	0.7855	0.6554	0.9781	0.4560	0.7524	0.9713
a_1	1.0580	1.1330	0.9540	1.2991	1.0797	0.9888
a_2	0.9950	0.9923	0.9998	0.9859	0.9939	0.9957
b_1	0.1203	0.1801	0.0596	1.0188	0.2469	0.2341
b_2	−0.0713	−0.0592	−0.0374	−0.1311	−0.0669	−0.0416
b_3	0.3202	0.1254	−0.0508	0.5736	0.1921	0.1370
b_4	0.2354	0.0996	0.2373	−0.3198	0.0374	0.0135
b_5	−0.2120	0.1230	0.0082	0.1047	0.1218	0.1497
MSE	1.294318	1.415953	1.153379	1.691296	1.908303	1.238136
R^2	0.977597	0.984431	0.977064	0.978322	0.982889	0.972638
n	211	708	3070	1413	1452	2360

[a] a_0, a_1, a_2, b_1, b_2, b_3, b_4, and b_5, parameter estimates; MSE, mean square error.

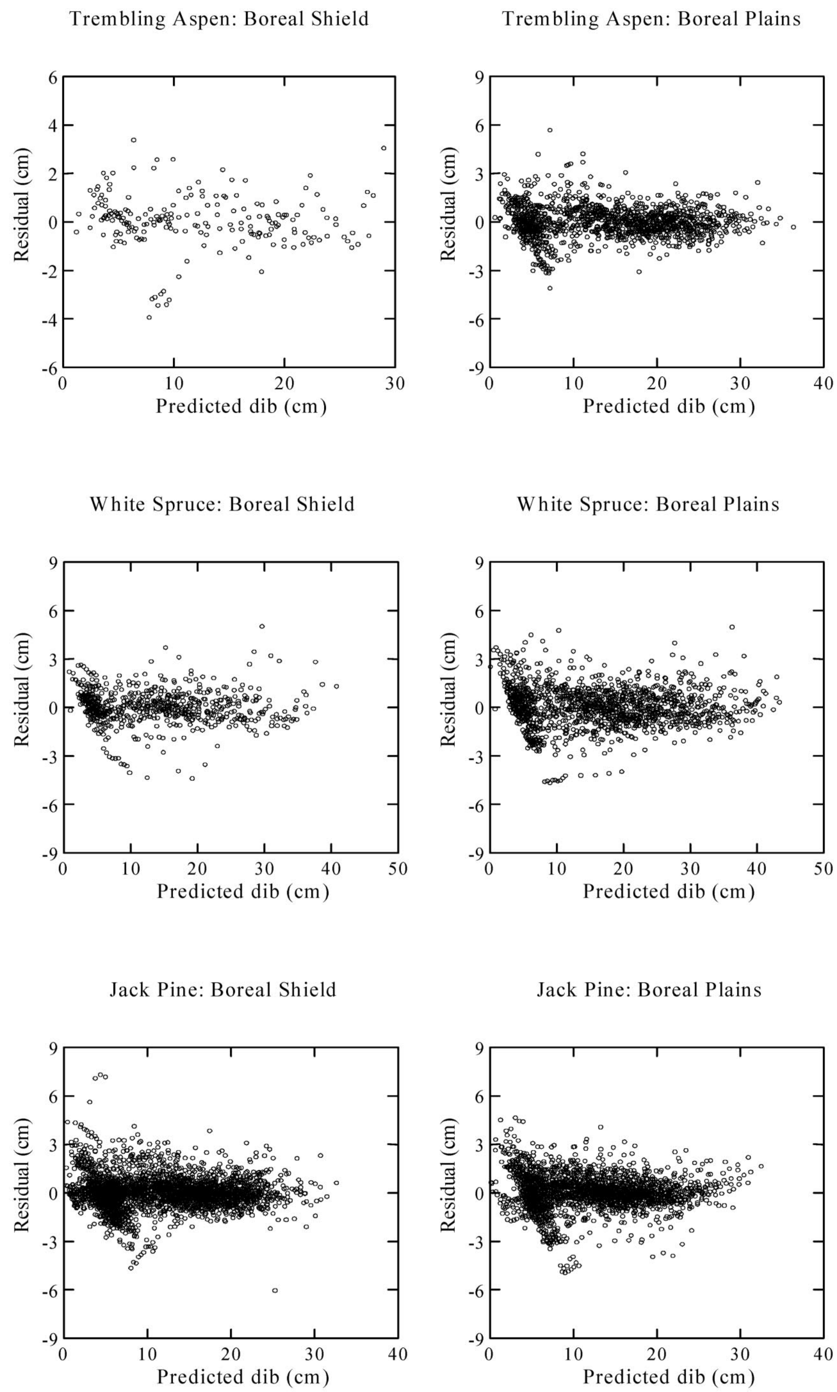

Figure 3. Residual plots of the ecozone-specific taper equations for trembling aspen, white spruce, and jack pine.

statistics (e.g., R^2 and MSE) were comparable with ours. Gal and Bella (1994) tested three equations, including those developed by Demaerschalk and Kozak (1977), Kozak (1988), and Hilt (1980) (modified from Bruce et al. [1968]) using 12 tree species in Saskatchewan. They found that Kozak's equation performed the best. Huang et al. (1999) demonstrated that the equation behaved well in predicting dib, GTV, merchantable height, and merchantable volume. Huang et al. (2000) also noted that the Kozak's equation was flexible, easy to use, and readily adaptable to any species.

We have detected a statistical difference in stem form between the two ecozones in Manitoba for trembling aspen, white spruce, and jack pine. These differences agree with those observed by Huang (1994) and Wang and Huang (2000) in Alberta. Huang (1994) examined differences of stem form among natural subregions in Alberta. Natural subregions are derived using a different ecological classification than that used for ecozones. Differences in stem form among natural subregions in Alberta were found for trembling aspen, balsam poplar, black spruce, balsam fir, white spruce, and lodgepole pine. Wang and Huang (2000) observed differences in height growth patterns of white spruce between two groups of natural subregions that occur in two separate ecozones. They attributed the difference in height growth patterns to the differences in climate between the ecozones.

Regardless of species, the GTV estimates derived from the Boreal Shield ecozone taper equations were, on average, less than those

Table 5. Comparison of GTV estimates derived from the ecozone-specific and provincial taper equations for trembling aspen, white spruce, and jack pine.[a]

Species	D (cm)	H (m)	GTV (m^3)		
			Boreal Shield	Boreal Plains	Provincial
Trembling aspen					
Small	12	13	0.0649 (1.4%)	0.0618 (−3.4%)	0.0640
Medium	20	21	0.2883 (−2.8%)	0.3046 (2.7%)	0.2965
Large	28	26	0.6757 (−3.5%)	0.7186 (2.6%)	0.7005
White spruce					
Small	16	13	0.1168 (−2.2%)	0.1217 (1.9%)	0.1194
Medium	25	20	0.4331 (−1.5%)	0.4432 (0.8%)	0.4396
Large	35	25	0.9929 (−1.4%)	1.0118 (0.5%)	1.0071
Jack pine					
Small	10	12	0.0451 (−4.0%)	0.0497 (5.7%)	0.0470
Medium	20	18	0.2649 (−2.1%)	0.2786 (2.9%)	0.2707
Large	30	22	0.7009 (1.1%)	0.6901 (−0.4%)	0.6932

Values in parentheses represent the percent difference compared with the provincial estimates. D, dbh outside bark; H, total tree height; GTV, gross total volume.

Table 6. Comparison of GTV estimates derived from Alberta (Huang 1994), Saskatchewan (Gal and Bella 1994), and Manitoba (this study) provincial taper equations.[a]

Species	D (cm)	H (m)	GTV (m^3)		
			Alberta	Saskatchewan	Manitoba
Balsam poplar					
Small	20	18	0.2104 (−6.5%)	0.2284 (1.5%)	0.2251
Medium	25	21	0.3800 (−6.2%)	0.4107 (1.3%)	0.4053
Large	30	24	0.6215 (−1.4%)	0.6654 (5.5%)	0.6305
Trembling aspen					
Small	12	13	0.0554 (−13.4%)	0.0697 (8.9%)	0.0640
Medium	20	21	0.2460 (−17.0%)	0.3175 (7.1%)	0.2965
Large	28	26	0.5887 (−16.0%)	0.754 (7.6%)	0.7005
White spruce					
Small	16	13	0.1139 (−4.6%)	0.1339 (12.1%)	0.1194
Medium	25	20	0.4190 (−4.7%)	0.4864 (10.6%)	0.4396
Large	35	25	0.9937 (−1.3%)	1.1527 (14.5%)	1.0071
Black spruce					
Small	10	12	0.0414 (−7.4%)	0.0489 (9.4%)	0.0447
Medium	18	20	0.2233 (−6.5%)	0.2667 (11.7%)	0.2387
Large	25	25	0.5227 (−3.7%)	0.6229 (14.8%)	0.5427
Jack pine					
Small	10	12	0.0427 (−9.1%)	0.0423 (−10.0%)	0.0470
Medium	20	18	0.2513 (−7.2%)	0.2709 (0.1%)	0.2707
Large	30	22	0.6558 (−5.4%)	0.7139 (3.0%)	0.6932

Values in parentheses represent the percent difference compared to the Manitoba provincial estimates. D, dbh outside bark; H, total tree height; GTV, gross total volume.

from the Boreal Plains ecozone equations, which may be attributed to a less favorable climate and poorer site conditions in the Boreal Shield ecozone. The climate of the Boreal Shield ecozone is slightly colder, with lower accumulated degree-days values (Environment Canada 2004a, Environment Canada 2004b). Soils in the Boreal Shield ecozone are more rocky, more sandy, and shallower than those of the Boreal Plains ecozone (Zoladeski et al. 1995). Rocky and sandy soils are less favorable to tree growth than the deep clay-loam soils of the Boreal Plains ecozone. Soil and site conditions may be more influential than climate on stem form, since the magnitude of difference in climate between the two ecozones is marginal. These less favorable climate and site conditions resulted in slower growth in the Boreal Shield ecozone, which, in turn, resulted in a greater height-to-diameter growth ratio (i.e., faster stem taper). Trees on these poorer sites produce less photosynthate. Since allocation of photosynthate to height growth is of higher priority than diameter growth, trees on the poorer sites, with limited photosynthate, possess less diameter increment (Oliver and Larson 1996). For a given tree size, faster stem taper yields less stem volume. Consequently, smaller GTV estimates were observed in our study for trees in the Boreal Shield ecozone.

In contrast to trembling aspen, white spruce, and jack pine, no statistical difference in stem form was observed for black spruce between the two ecozones in Manitoba. The results disagree with Huang (1994) who observed differences of black spruce stem taper among natural subregions of Alberta. The differences found in Alberta, however, may be largely related to altitudinal differences. Of the two groups of natural subregions with significant differences in stem form, one is located close to the mountains, whereas the other is located east of the mountains in the central portion of the province (Huang 1994). Unlike in Alberta, there is hardly any difference in elevation between the two ecozones in Manitoba. Because black spruce typically grows on poorly drained sites with deep organic soil, little difference in soil condition is expected between the two ecozones.

Although statistical differences in stem form were found between ecozones for trembling aspen, white spruce, and jack pine, accounting for this difference appears to be of little practical importance. On average, the Boreal Shield and Boreal Plains ecozone-specific taper equations produced GTV estimates within −1.7 percent and 1.5 percent, respectively, of those obtained from the provincial equations. Huang (1994) observed differences in stem taper among natural subregions of Alberta. According to our calculations, regardless of species, GTV estimates derived from his regionalized equations were also similar (<3 percent difference) to those derived from the provincial equations. Using ecozone-specific taper equations increases the complexity of forest management and appears to be unjustified when accounting for a difference of less than 3 percent. Given such a small difference, our results suggest that ecologically based taper equations may not be necessary and that one provincial taper equation for each species should be satisfactory for volume estimation and forest management planning.

The GTV estimates from the provincial taper equations for Alberta, Saskatchewan, and Manitoba exhibit greater variability than estimates within a province at the ecozone and natural subregion level. On average, the Alberta and Saskatchewan provincial taper equations produced GTV estimates of −7.4 percent and 6.5 percent, respectively, compared with those from the Manitoba provincial equations. Although the differences of stem form within provinces appeared to be marginal, the between province variation of stem form is much larger and should be accounted for. Therefore, a provincial stem taper equation for each species and province should be satisfactory for volume estimation.

Some of the taper equations developed in our study were based on data obtained from fewer than the 60 trees recommended by Kozak. When additional data become available, these equations should be updated. It is desirable that new data be collected outside the current range of dbh and total tree height measurements to expand the application of the equations. In this study, we have summarized the data used for the development of each taper equation. It is advised that application of our taper equations should not proceed outside the range of dbh and total tree height data used in the study. In addition, the provincial balsam poplar taper equation was developed solely with data from the Boreal Plains ecozone. Therefore, caution should be exercised when applying the equation across the entire province.

Conclusions

Kozak's variable-exponent taper equation provided a good fit to the data for all the species. The provincial and ecozone-specific taper equations developed in this study explained more than 97 percent of the total variation in dib, indicating that Kozak's variable-exponent model is appropriate for the five Manitoba tree species tested.

Differences in stem form were observed for trembling aspen, white spruce, and jack pine between the Boreal Shield and Boreal Plains ecozones in Manitoba. However, GTV estimates derived from these ecozone-specific taper equations differed only by <2 percent from GTV estimates derived from the provincial taper equations. Given such a small difference, a single provincial stem taper equation for each species may be satisfactory for estimating stem volume.

Although the regional difference in stem form was marginal within a province, a difference of approximately 7 percent in GTV estimation was found when comparing the provincial taper equations from Alberta, Saskatchewan, and Manitoba. This suggests that stem form varies more at a greater spatial scale and that using taper equations developed in other provinces should be avoided.

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